Berkeley Math Circle: Monthly Contest 4

Due January 20, 2021

Instructions

- This contest consists of seven problems of varying difficulty. Problems 1–4 comprise the *Beginner Contest* (for grades 8 and below) and Problems 3–7 comprise the *Advanced Contest* (for grades 9–12). Contest 4 is due on January 20, 2021.
- Begin each submission with your name, grade, school, BMC level, the problem number, and the contest number on every sheet. An example header:

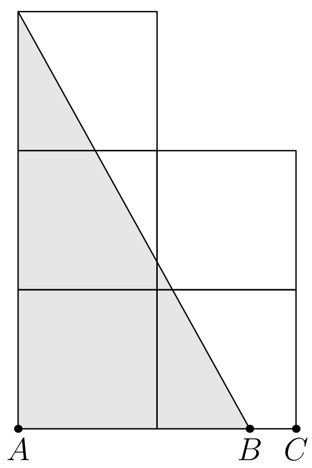
  BMC Monthly Contest 4, Problem 2
  Evan O'Dorney
  Grade 3, BMC Beginner
  from Springfield Middle School, Springfield

  Submit **different problems on different pages** as they are graded separately.
- Each problem is worth seven points; to receive full points all results must be completely proven. Include all relevant explanations in words and all intermediate calculations; answers without justification will receive little or no credit. Submit solutions to as many problems as you can since partial credit will be awarded for sufficient progress.
- Remember you are not allowed to talk to anyone else about the problems, but you may consult any book you wish. See the BMC website at `http://mathcircle.berkeley.edu` for the full rules.

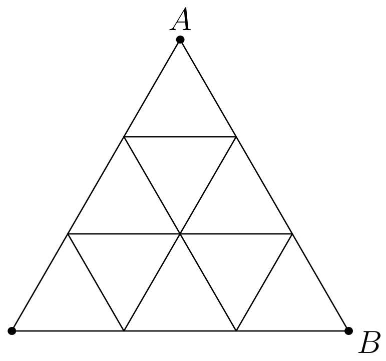
Enjoy solving these problems and good luck!

Problems for Contest 4

1. The figure below consists of five congruent squares. If the area of the shaded triangle equals the area outside the shaded triangle, calculate the ratio AB/BC.



2. Suppose you have an equilateral triangle divided into 9 smaller equilateral triangles as shown, with the bottom side horizontal. Starting from the top corner labeled A, you must walk to the bottom right corner labeled B, and are only allowed to take steps along the edges down to the left, down to the right, or horizontally to the right. Determine the number of possible paths.



3. Let gcd mean the greatest common divisor of two numbers and lcm their least common multiple. Suppose the three numbers A, B, C satisfy

$$\begin{aligned} \gcd(A, B) = 2, \quad \operatorname{lcm}(A, B) = 60 \\ \gcd(A, C) = 3, \quad \operatorname{lcm}(A, C) = 42. \end{aligned}$$

Determine the three numbers.

4. Aerith timed how long it took herself to solve a BMC monthly contest. She writes down the elapsed time as days:hours:minutes:seconds, and also simply as seconds. For example, if she spent 1,000,000 seconds, she would write down 11:13:46:40 and 1,000,000.

Bob sees her numbers and subtracts them, ignoring punctuation; in this case he would get

$$11134640 - 1000000 = 10134640.$$

What is the largest number that always must divide his result?

5. Are there integers a and b for which $a^2 = b^{15} + 1004$?

6. In a convex n-sided polygon, all the diagonals are drawn and no three of them pass through a point. Find a formula for the number of regions formed inside the polygon.

7. Let P be a point on segment BC of triangle $\triangle ABC$. Let O_1 and O_2 be the respective circumcenters of $\triangle ABP$ and $\triangle ACP$. Given $BC = O_1O_2$, show that some angle of $\triangle ABC$ is more than $75°$.