Discrete Calculus

CALCULUS is a PROMINENT part of Mathematics that helps to study behavior of functions, find areas and volumes, solve equations, and do many other things.

Discrete Calculus is its much less famous relative in the world of *functions of discrete variables*. It has its own versions of derivatives, integrals, and related formulas. It is very useful not only for the ordinary Calculus, but also for solving problems like those listed below, that look very far from the traditional range of applications of Calculus. Try some of them now.

1. Find the next few terms of the sequence
   (a) $1, 3, 8, 16, 27, \ldots$
   (b) $1, 3, 8, 17, 31, 51, \ldots$
   (c) $1, 1, 2, 5, 13, 31, \ldots$

2. Find a formula for the nth term of the above sequences.

3. Find a polynomial $p(x)$ of degree four (i.e. $p(x) = ax^4 + bx^3 + cx^2 + dx + f$) such that
   $p(0) = 1,\ p(1) = 2,\ p(2) = 4,\ p(3) = 8,$ and $p(4) = 16.$

4. (AIME 1989) Assume that $x_1, x_2, \ldots, x_7$ are real numbers such that
   $x_1+4x_2+9x_3+16x_4+25x_5+36x_6+49x_7 = 1$, $4x_1+9x_2+16x_3+25x_4+36x_5+49x_6+64x_7 = 12$, and $9x_1 + 16x_2 + 25x_3 + 36x_4 + 49x_5 + 64x_6 + 81x_7 = 123.$
   Find the value of $16x_1 + 25x_2 + 36x_3 + 49x_4 + 64x_5 + 81x_6 + 100x_7$.

5. Find the following sums
   (a) $1 + 2^2 + 3^2 + 4^2 + \ldots + n^2$;
   (b) $1 + 2^3 + 3^3 + 4^3 + \ldots + n^3$;
   (c) $1 + 2 \cdot 3 + 3 \cdot 3^2 + \ldots + n \cdot 3^{n-1}$.
   (d) $\dfrac{1}{1 \cdot 2 \cdot 3} + \dfrac{1}{2 \cdot 3 \cdot 4} + \ldots + \dfrac{1}{n \cdot (n+1) \cdot (n+2)}$.

6. Two straight cuts can divide a round pizza into 3 or 4 pieces. What is the maximum number of pieces into which a round pizza can be divided using 10 straight cuts?

7. On the circumference of a circle, n points in general position are marked, and every pair of points is connected by a chord. For $n = 2,\ 3,\ 4$, these chords divide the circle into 2, 4, and 8 regions respectively. Into how many regions will the circle be divided for $n = 5$? And for an arbitrary n?