BERKELEY MATH CIRCLE

Problem Solving Techniques:

Solutions,
Not Answers

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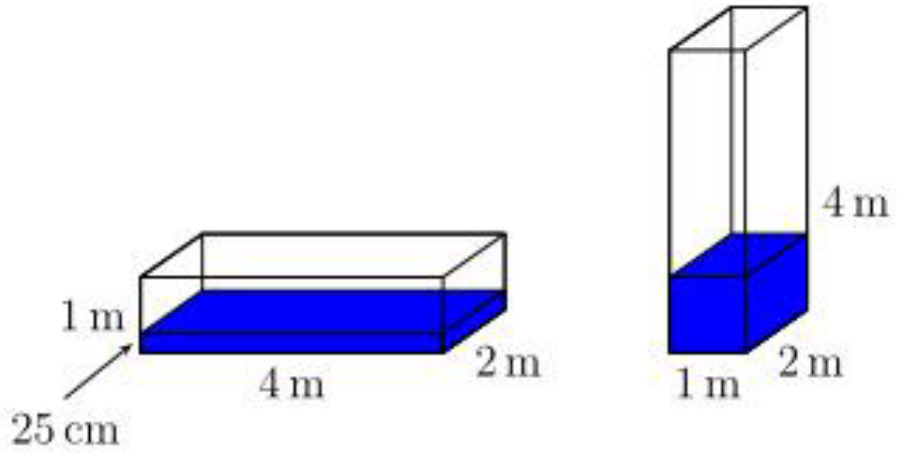
A water tank with a rectangular base has the dimensions of 1m x 2m x 4m. It contains water to a depth of 25cm, as shown in the first picture. The tank is turned so that a 1m x 2m face is now the base, as shown in the second picture. What is the depth of the water now?



SOLUTION

The volume of the water does not change, therefore the calculation for the volume in each container is identical.

First Picture Volume = (4m)(2m)(25cm)= 4 x 2 x 0.25 = 2
Second Picture Volume: (1m)(2m)(x) = 2x
$2x = 2 \rightarrow x = 1$ meter

The depth of the water in the second container is 1 meter.

A marching band has 150 members. One day only part of the marching band shows up, but nobody wants to take the time to count how many members there are present (so lazy!). They first line up in rows with 5 members each, and there is one left over member. Then they try rows with 6 members each, but there is still one leftover member. Then they try rows of 7, but there are two leftover members. How many members should line up in each row so that no members will be left over, and how many rows are created when doing so? A row has to have more than one member.

SOLUTION

There is 1 person left over when rows of 5 and 6 members are formed. If we removed this one person, the resulting number (30) would be both a multiple of 5 and 6. Therefore, the original number of members must be a multiple of 30, but more than 30 total members (30 + 1 = 31 at the very least). There cannot be more than 150 people, so all possible values are:

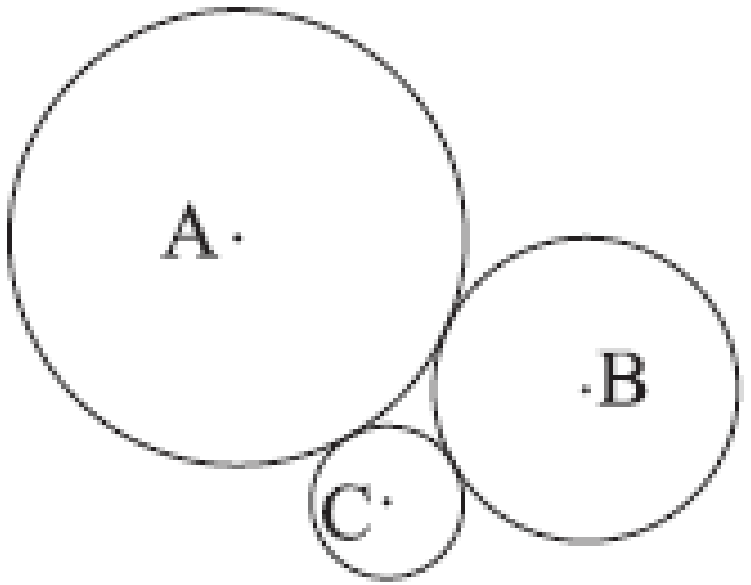
31, 61, 91 and 121

The final value must also give the remainder of 2 when divided by 7. From the above, we have:

31/7 = 4R3
61/7 = 8R5
91/7 = 13
121/7 = 17R2

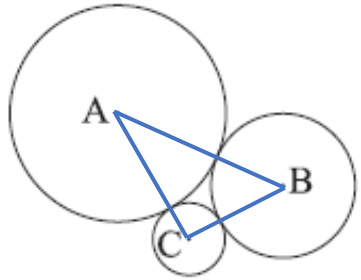
Therefore, the marching band should form 11 rows with 11 musicians per row.

Three mutually externally tangent circles with centers A, B, and C have the radii of 3, 2, and 1, respectively. What is the area of the triangle ABC?



SOLUTION

First, we connect A, B & C to form a triangle. Based on the radii given, we see that:



AB = 5 (3 +2)
BC = 3 (2 +1)
AC = 4 (3 + 1)

The values of the triangle satisfy the criteria to be classified as a right triangle, via the Pythagorean Theorem (not just because you think it looks like a right triangle!):

$a^2 + b^2 = c^2 \rightarrow 3^2 + 4^2 = 5^2 \rightarrow 9 + 16 = 25 \rightarrow 25 = 25$

Therefore, the legs of the right triangle have lengths 3 and 4, while it's hypotenuse has length 5.

Area Triangle = (½)(b)(h) = (½)(leg 1)(leg 2) = (½)(3)(4) = (½)(12) = 6

The area of the triangle ABC is 6 square units.

A rectangle is given with length l and width w, where the length > width. One of them is increased by 20%, while the other is decreased by 20%.

Does the area of the rectangle increase, decrease or stay the same, or does it depend on the given set of values?

Justify your conclusion with logical reasoning and maybe some math ;)

The area of any rectangle is equal to the length times the width of that rectangle. For any given set of dimensions for a rectangle, x and y, the area is:

$A = xy$

Since x and y are multiplied by each other, it does not matter which one is increased by 20% and which one is decreased by 20% (go commutative property!). The new values to use are:

Length = (1.2)(x)
Width = (0.8)(y)

Area = [(1.2)(x)][(0.8)(y)] = (0.96)(xy)

Our new area will ALWAYS be 4% smaller than the original area.