# BERKELEY MATH CIRCLE

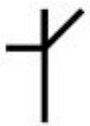
## Problem Solving Techniques:

## Math Olympiad
## Selections I

**Instructor: Patricio Angulo**

# Alternate Version (a little harder)

Cistercian numerals were used in the early thirteenth century. Any integer from 1 to 99 can be represented by a single glyph formed by combining two of the glyphs shown below.


The glyph for 24 looks like  , the glyph for 81 looks like  , and the glyph for 93 looks like  . What does the glyph for 45 look like?

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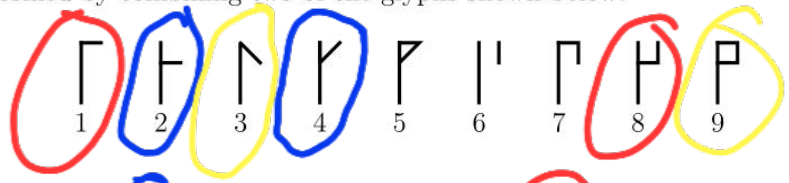
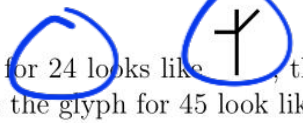



The glyph for 24 looks like  , the glyph for 81 looks like  , and the glyph for 93 looks like  .
What does the glyph for 45 look like?

24 is a combo of the 2 and 4 symbols, BUT the 2, in the tens place, is written in reverse. The same holds for the symbols for 81 and 93.

Following this pattern, the 4 of 45 is reversed to look like Y. The 5 of 45 is written as shown (P)

∴ the glyph for 45 is Y + P = 

Each year, the third Thursday in March is called Kangaroo Day. The dates for the next six Kangaroo Days are listed below, but two of them are listed incorrectly. Which dates are wrong and why?

A) March 15th, 2029
B) March 16th, 2028
C) March 14th, 2027
D) March 20th, 2031
E) March 22nd, 2030
F) March 19th, 2026

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1 Week = 7 Days = 1 Thursdays
2 Weeks = 14 Days = 2 Thursdays
3 Weeks = 21 Days = 3 Thursdays
14 and under = incorrect
Over 21 = incorrect

1, 8, 15
2, 9, 16
3, 10, 17
4, 11, 18
5, 12, 19
6, 13, 20
7, 14 21

If March 1st is the 1st Thursday, then the next two Thursdays are on 8th & 15th
following this pattern, we have (2, 9, 16) & (3, 10, 17) & (4, 11, 18) & (5, 12, 19) & (6, 13, 20) & (7, 14, 21). Possibilities for the 3rd Thursday are therefore 15, 16, 17, 18, 19, 20 & 21. ∴ C & E are incorrect.

What is the value of the following?

$$\frac{7777^2}{(5555)(2222)}$$

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$$\frac{7777^2}{(5555)(2222)}$$

$$= \frac{(7777)(7777)}{(5555)(2222)}$$

$$= \frac{(7\times1111)(7\times1111)}{(5\times1111)(2\times1111)}$$

Cancellations possible!

$7777 = (7\times1111)$

$$= \frac{7\times7}{5\times2} = \boxed{\frac{49}{10}}$$

A square of numbers is taken out from a multiplication table. Only one number is visible. The integers $x$ and $y$ are both positive, and $x$ is greater than $y$. What is the value of $x$ and $y$, and what values go in the empty boxes?

|  | x | x + 1 |
|---|---|---|
| y | xy | (y)(x+1) |
| y + 1 | x(y+1) | 77 |

| | 10<br>x | 11<br>x + 1 |
|---|---|---|
| 6<br>y | 60 | 66 |
| 7<br>y + 1 | 70 | 77 |

$(x+1)(y+1) = 77$

$77 \times 1 = \emptyset$

$7 \times 11$

$y = 0$ and both $x$ and $y$ are positive

$7 \quad 11$

$x \quad y$

$x+1 > y+1$

$\therefore \quad x+1 = 11 \rightarrow x = 10$

$y+1 = 7 \rightarrow y = 6$