BERKELEY MATH CIRCLE

Problem Solving Techniques:

Math Olympiad
Selections II

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This year, a grandmother, her daughter, and her granddaughter can say that the sum of their ages is 100. In what year was the granddaughter born if each of their ages is a power of 2?

We need to explore powers of 2 since their ages are given as powers of 2.

$2^x + 2^y + 2^z = 100$

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$

We need 3 of these to add up to 100.
$\therefore 2^7$ is out.

We want 3 single digits to add to 200 (also, just want 3 #s = 100)
It works for 4, 32, 64 $\Rightarrow 4+32+64=100$

$\therefore$ The granddaughter is <u>now</u> 4 years old, and thus she was born in 2021.

Some glasses are stacked on top of each other as shown. A stack of 8 glasses is 42 cm high, and a stack of 2 glasses is 18 cm high. How high is a stack of 6 glasses?

Comparing stacks, $42/8 \neq 18/2$, ∴ there is other height to worry about.

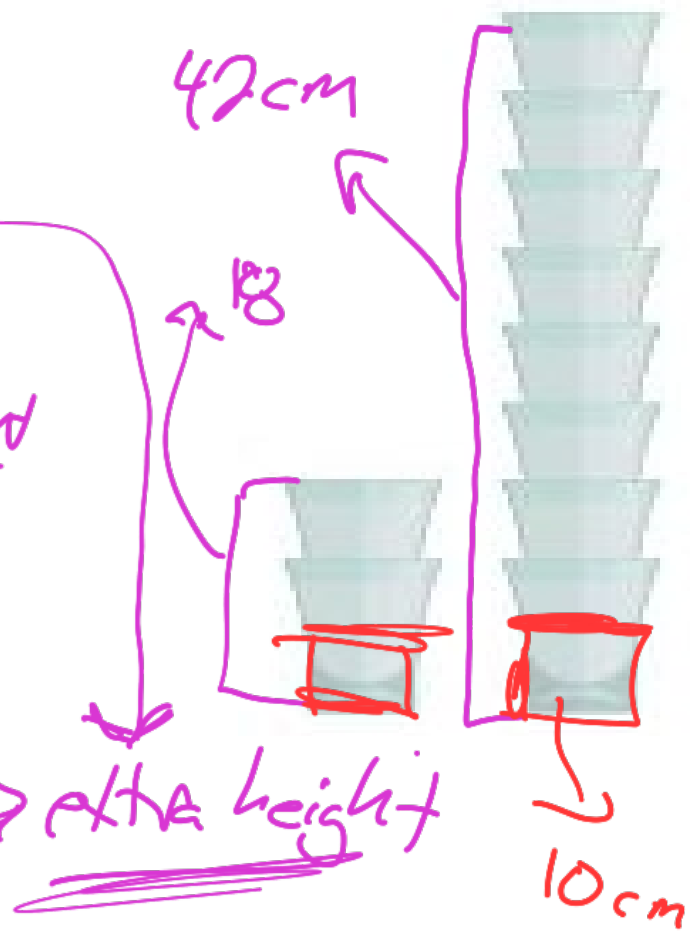
$42 - 18 = 24$ cm ⇒ 6 cups w/o the bottom height

So, $24/6 = 4$ cm per top of glasses

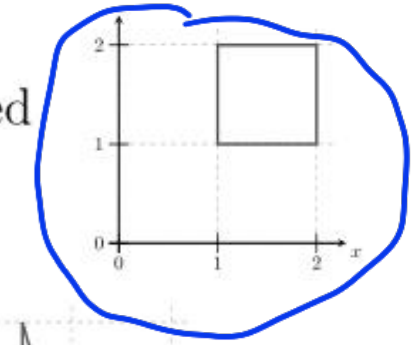
Looking at the 2 cup stack ⇒ $18 = (2 \cdot 4) + x$ → bttm height

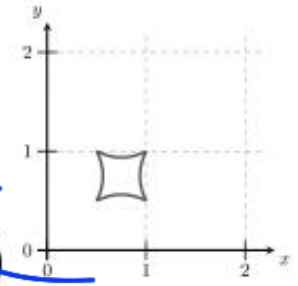
$x = 10$

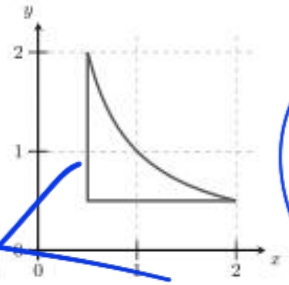
∴ a stack of 6 glasses $= 4 \cdot 6 + 10 = \boxed{34\text{ cm}}$

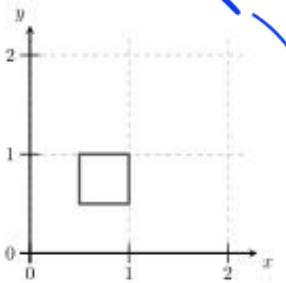


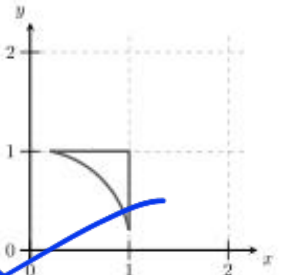
A square lies in a coordinate system as shown. Each point (x, y) on the square is moved to $\left(\frac{1}{x}, \frac{1}{y}\right)$. What will the resulting figure look like?

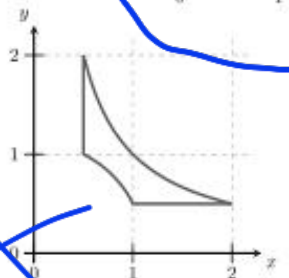


(A) 

(B) 

(C) 

(D) 

(E) 

Don't Use Coordinates!!

- B and E = bigger than original square → not necessarily!
- Point to Point = a straight line ⟹ 
- 4 points in original = 4 pts in new → ~~B~~, ~~D~~

The license plate of Zvezda's car fell off (oh no!). She put it back upside down, but luckily this did not make any difference. Which of the following could be Zvezda's license plate and why? The correct answer needs to explain why four choices are incorrect and only one can be the correct answer (5 sentences).

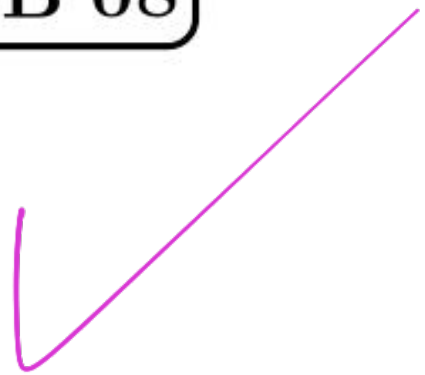
(A) 04 NSN 40

(B) 60 HOH 09

(C) 80 BNB 08

(D) 03 HNH 30

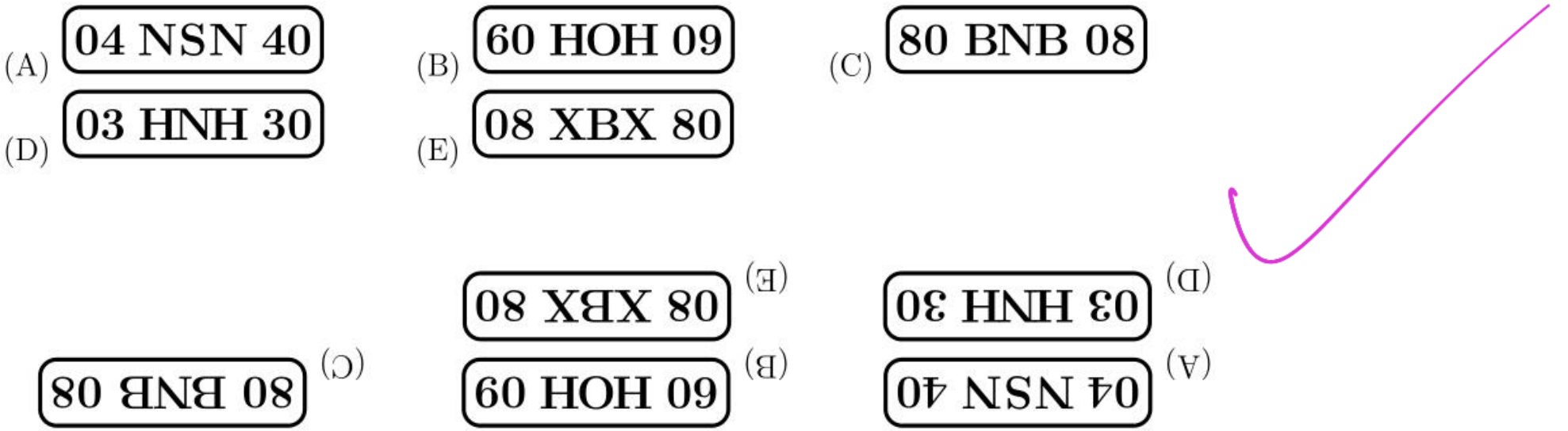
(E) 08 XBX 80



ANSWER

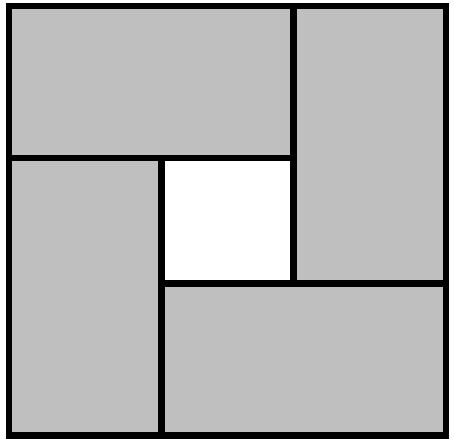
The initial reasoning here is that we need to think (and see) what all of these license plates look like upside down. The one that produces a match to the original license plate is the answer, and **symmetry** is key:

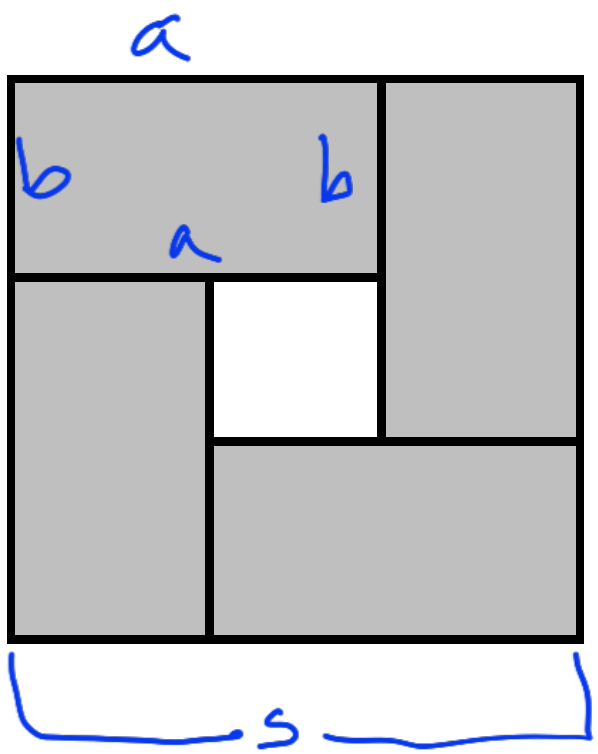
- A is not the answer as only the 0's look the same upside down, and maybe the S. All others do not look the same.
- **B is the answer as all letters, numbers, and the order they are in look the same upside down.**
- C is not the answer as the 0's look identical, the 8's are close, but the BNB does not look the same upside down.
- D is not the answer as the 0's look identical, the H's are close, but the 3 and N do not look the same upside down.
- E is not the answer as the 0's look identical, the 8's and X's are close, but the B does not look the same upside down.



The diagram shows four identical rectangles placed inside a square. The perimeter of each rectangle is 16 cm. What is the perimeter of the big square?

Must show all work, including any equations used!





Perimeter Rectangle = 16cm

$\therefore 2a + 2b = 16 \Rightarrow a + b = 8cm$ (÷ 2)

One edge of the square = $s = a + b = 8cm$

Perimeter Square = $4s = 4 \cdot 8 = 32$

The perimeter of the square is 32cm