BMC Fall 2023 Int I

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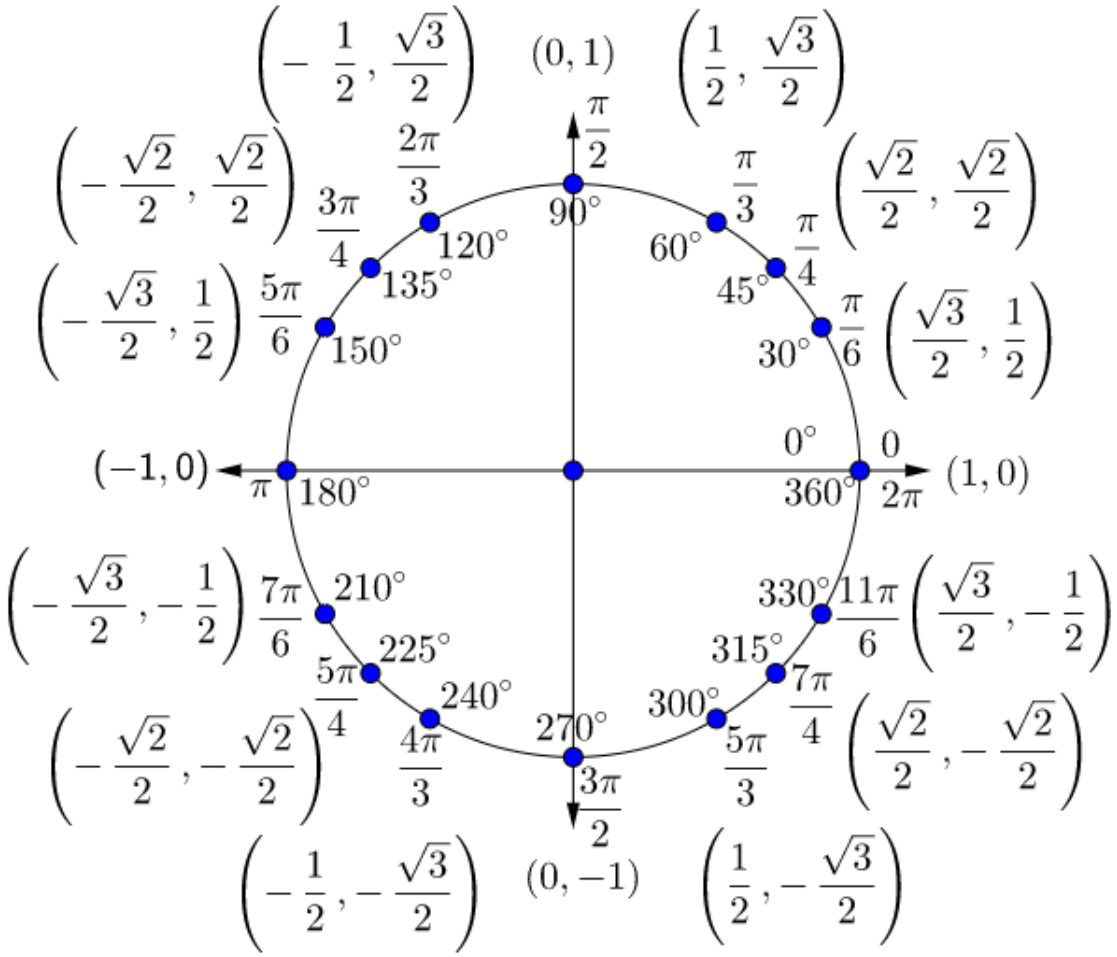
1 Trigonometry

Trigonometry Problems



1. For right ΔABC with side a = 20, c = 21, and $B = 90°$ find side b, $cos(C)$, angle C, and angle A.
   (Note: there is a naming convention in mathematics where you label the angles of a triangle with capital letters, starting on the left most point, traveling counter-clockwise, and enumerating in alphabetical order. Lowercase letters refer to the side measures and are directly opposite of their angles).

2. For right ΔABC with side a = 30, c = 16, and $B = 90°$, find side b, $sin(A)$, $cos(A)$, angle C, and angle A.

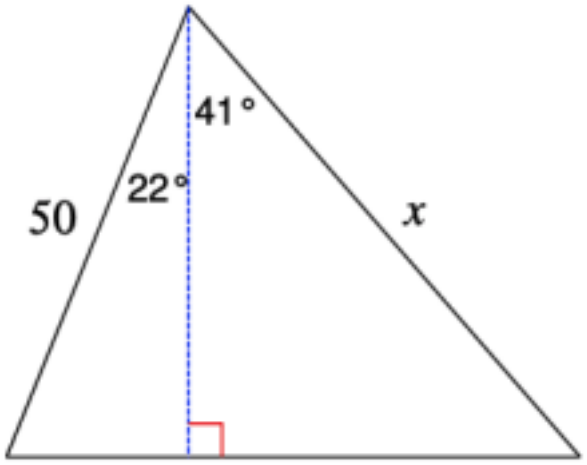
3. For right ΔABC with side a = 40, c = 9, and $B = 90°$, find side b, find $sin(A)$, $cos(A)$, $cos(C)$, $sin(C)$ and compare your answers. What conclusions can you make?

4. For right ΔABC with side a = 30, c = 40, and $B = 90°$ find side b, $sin(A)$, $cos(A)$ and $tan(A)$, angle C, and angle A.

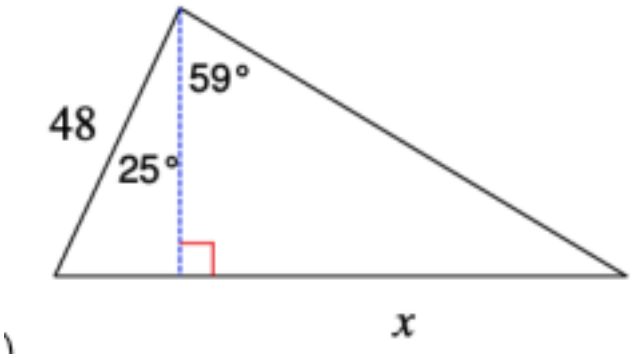
5. Find all 6 trigonometric functions, and the measure of angle θ if the opposite side of the angle is 5 and the hypotenuse is 8.

6. Find the length of the side labeled x.

   (a) Triangle 1



   (b) Triangle 2



7. The angle of elevation of the top of a tree is 30° from a point 28 ft away from the foot of the tree. Find the height of the tree rounded to the nearest foot.

8. A ladder with its foot on a horizontal flat surface rests against a wall. It makes an angle of 70° with the horizontal. The foot of the ladder is 4 ft from the base of the wall.

   (a) Draw and label a sketch of the situation.

   (b) Find the height of the point where the ladder touches the wall.

   (c) Find the length of the ladder.

9. From the top of the Empire State building 1250ft, the angle of depression to a car on ground level is 36° Draw a diagram and find the distance from the base of the building to the car.

10. A passenger in a plane passenger in an airplane sees two towns directly to the left of the plane. Find d, x and y as shown in the diagram.

11. A man on the deck of a ship is 15 ft above sea level. He observes that the angle of elevation of the top of a cliff is 70° and the angle of depression of its base at sea level is 50° .

    (a) Sketch and label a diagram

    (b) Find the height of the cliff

    (c) Find the distance from the ship to the base of the cliff.

12. A 14-foot ladder is used to scale a 13-foot wall. At what angle of elevation must the ladder be situated in order to reach the top of the wall?

13. The angle of elevation of the top of a cliff from the point Q on the ground is 28°. On moving a distance of 20m towards the foot of the cliff the angle of elevation increases to $x°$ If the height of the cliff is 37m find $x°$

14. The point P is on the terminal side of angle . Sketch a reference triangle. Evaluate the six trigonometric functions for θ Write "undefined" if the trig function has no value.

    (a) $P(4, 3)$

    (b) $P(22, -22)$

    (c) $P(0, -4)$

    (d) $P(-2, 0)$

15. Sketch a reference triangle in the appropriate quadrant for the given info.

    (a) Given: $cos(\theta) = \frac{2}{3}$, and $cot(\theta) > 0$, find $sin(\theta)$ and $tan(theta)$.

    (b) Given: $sin(\theta) = \frac{1}{4}$, $tan(\theta) < 0$, find $cos(\theta)$ and $tan(\theta)$.

    (c) Given $sin(\theta) = \frac{-2}{5}$, and $cos(\theta) > 0$, find $tan(\theta)$ and sec(θ)

16. Sketch a reference triangle and find the angle for each inverse trig function. No calculator.

    (a) $sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

    (b) $cos^{-1}\left(\frac{-1}{2}\right)$

    (c) $sin^{-1}\left(\frac{-1}{2}\right)$

17. Draw the triangle represented by the following expressions. Rewrite each expression so that it does not involve trigonometric functions or inverse trigonometric functions.

    (a) $cot(sin^{-1}(x))$

    (b) $cos(tan^{-1}(2x))$

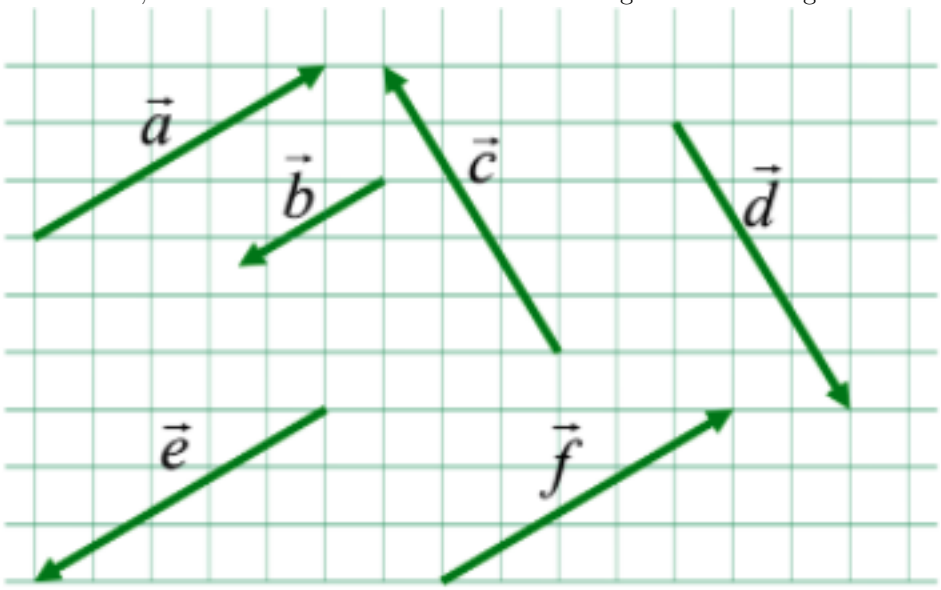
    (c) $sec(cos^{-1}(x))$

    (d) $csc(tan^{-1}(x))$

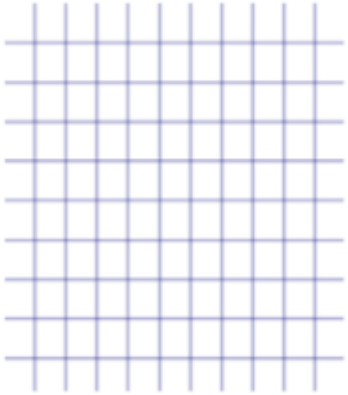
2 Vectors Part 1 and Part II

Vector Problems

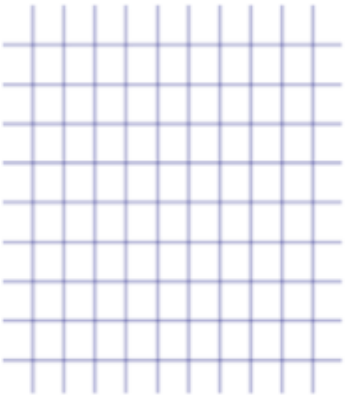
1. For each vector in the diagram, write the vector in component form, find the magnitude of the vector, and find the direction of the vector using a standard angle.



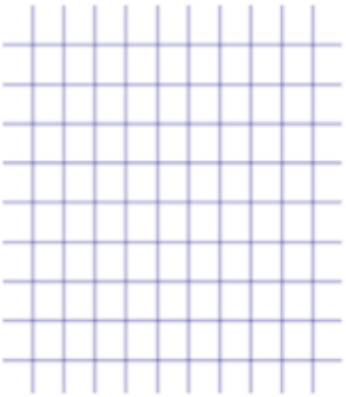
2. Sketch the vector and determine the component form
   (a) $|\vec{p}| = 10$, direction is $20°$
   (b) $|\vec{q}| = 8$, direction is $120°$
   (c) $|\vec{r}| = 40$, direction is $-60°$
3. Let $\vec{v} = \langle -2, 5 \rangle$ and let $\vec{w} = \langle 3, 4 \rangle$, find the component form of each of the following vectors.
   (a) $2\vec{v}$
   (b) $\vec{w} + \vec{v}$
   (c) $\vec{v} + \vec{w}$
   (d) $\vec{v} - \vec{w}$
   (e) $\vec{w} - \vec{v}$
   (f) $2\vec{w} - 3\vec{v}$
4. Show the vector addition and vector subtraction graphically, in whichever method you choose.
   (a) $\vec{v} = 4\vec{i} + 2\vec{j}$
   $\vec{u} = 2\vec{i} - 3\vec{j}$
   $\vec{w} = \vec{v} + \vec{u}$



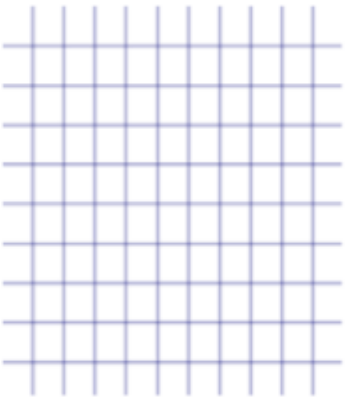
(b) $\vec{v} = 4\vec{i} + 2\vec{j}$
$\vec{u} = 2\vec{i} - 3\vec{j}$
$\vec{w} = \vec{v} - \vec{u}$



(c) $\vec{v} = \langle 2, -2 \rangle$
$\vec{u} = \langle -4, 1 \rangle$
$\vec{w} = 3\vec{v} + 2\vec{u}$



(d) $\vec{v} = \langle 5, -2 \rangle$
$\vec{u} = \langle 3, 2 \rangle$
$\vec{w} = 2\vec{v} - \vec{u}$

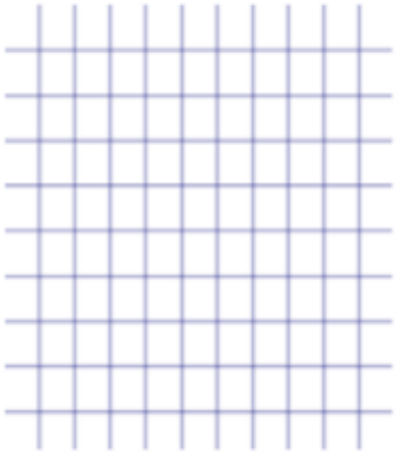


5. Graph the following points and write the component form for the vectors between each points. Then graph each vector, and graph the sum of all three.
$P(-1, 3)$, $Q(1, 5)$, $R(3, -2)$, $S(2, 8)$
$\overrightarrow{PQ} =$

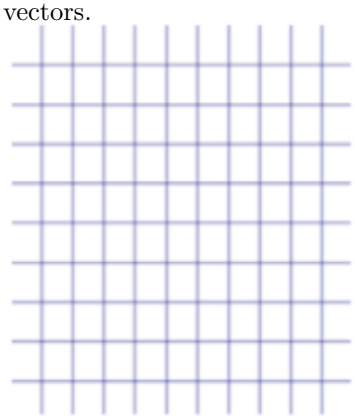
$\overrightarrow{QR} =$

$\overrightarrow{RS} =$

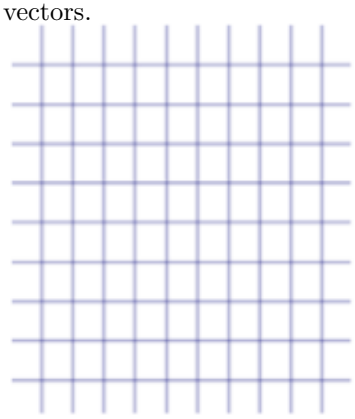
$\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} =$



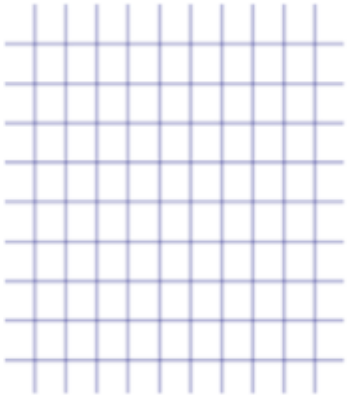
6. Calculate the angle between the 2 vectors $\vec{v} = \langle 3, -8 \rangle$ and $\vec{w} = \langle -2, -1 \rangle$.
7. Calculate the angle between the 2 vectors $\vec{v} = \langle -5, 2 \rangle$ and $\vec{w} = \langle 4, -10 \rangle$.
8. Calculate the angle between the 2 vectors $\vec{v} = \langle -7, 6 \rangle$ and $\vec{w} = \langle -7, 6 \rangle$.
9. Find "k" such that the 2 vectors are perpendicular: $\vec{v} = \langle 2, k \rangle$ and $\vec{w} = \langle 12, -6 \rangle$.
10. Calculate the angle between the 2 vectors $\vec{v} = \langle 5, 8 \rangle$ and $\vec{w} = \langle -5, -8 \rangle$.
11. Calculate the angle between the 2 vectors $\vec{v} = \langle 6, -4 \rangle$ and $\vec{w} = \langle -3, 2 \rangle$.
12. Find "k" such that the 2 vectors are parallel: $\vec{v} = \langle 2, k \rangle$ and $\vec{w} = \langle 8, -2 \rangle$.
13. Without calculating the actual angle value, how can you tell if the angle between 2 vectors will be acute? right? obtuse?
14. Find the vector projection of $\vec{v} = \langle -2, 6 \rangle$ onto $\vec{w} = \langle -3, -1 \rangle$. Sketch a diagram with all 3 vectors.



15. Find the vector projection of $\vec{v} = \langle 5, 1 \rangle$ onto $\vec{w} = \langle -3, -2 \rangle$. Sketch a diagram with all 3 vectors.



16. Find the vector projection of $\vec{v} = \langle 10, 2\rangle$ onto $\vec{w} = \langle -3, 15\rangle$. Sketch a diagram with all 3 vectors.



17. Taylor is sitting on a sled on the side of a hill inclined at 45°. The combined weight of Taylor and the sled is 170 pounds. What is the magnitude of the force required for Harrison to keep the sled from sliding down the hill?

18. A 2000-pound car is parked on a street that makes an angle of 12° with the horizontal ground.

    (a) Find the magnitude of the force required to keep the car from rolling down the hill.

    (b) Find the force perpendicular to the street.

19. Find the work done lifting a 2600-pound car 5.5 feet.

20. Find the work done lifting a 100-pound bag of potatoes 3 feet.

21. Find the work done by a force $\vec{F}$ of 12 pounds acting in the direction $\langle 1, 2\rangle$ in moving an object 4 feet from $(0, 0)$ to $(4, 0)$

22. The angle between a 75 pound force $\vec{F}$ and $\overrightarrow{AB}$ is 60° where $A = (-1, 1)$ and $B = (4, 3)$. Find the work done by $\vec{F}$ moving an object from A to B.

23. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides