BMC Intermediete I: Pigeons, Holes, and the Pigeonhole Principle

Felicia Lim | 2 October 2024

We learned three forms of the Pigeonhole Principle.

- If $k+1$ pigeons are put into k holes, some hole has at least two pigeons.
- If $(n-1)k+1$ pigeons are put into k holes, some hole has at least n pigeons.
- If n pigeons are put into k holes, some hole must have at least $\lceil \frac{n}{k} \rceil$ pigeons.

Use any of these three forms to solve the following problems.

1. I have a bag containing 3 types of chocolate. What is the smallest number of chocolates I need to pick from the bag to ensure that I picked 3 of the same flavor?
2. There are 39 million people in California. How many people can we guarantee have the same two-letter initials?
3. Prove that if 101 numbers from 1 to 1000 are chosen, at least 11 of them must have the same ones digit.
4. Nine students solved a total of 28 problems in a math olympiad. Each problem was solved by exactly one student. We know of one student who solved exactly one problem, two students who solved exactly two problems, and three students who solved exactly three problems. Prove that one student must have solved at least 5 problems.
5. Pick 7 points on the surface of a sphere. Show that at least 5 must lie on the same hemisphere of the sphere.
6. Given any 9 natural numbers $(1, 2, 3, \dots)$, show that it is possible to choose 5 whose sum is divisible by 5.
7. Prove that in a set of ten distinct 2 digit numbers, it is possible to select two nonempty disjoint subsets whose members have the same sum.
8. Come up with a problem that can be solved using the Pigeonhole Principle. Have a partner solve it.