BMC Beginners II
Modular Arithmetic II

Name: ________________
Date: ________________

Review

Euclidian Division
Find the remainder of:

12)13
12)10
12)-2

Clock Math
Calculate:

11 + 2 = __ (mod 12)
5 + 5 = __ (mod 12)
5 - 7 = __ (mod 12)

Equivalence (mod m)
Find the **additive** inverse of:

5 + __ = 0 (mod 12)
7 + __ = 0 (mod 12)
6 + __ = 0 (mod 12)

Using Equivalence

Is 279 + 15 divisible by 7?

279 = __ (mod 7)
15 = __ (mod 7)
279 + 15 = __ + __ = __ (mod 7)

What is 500 + 1000 (mod 1021)?

500 = __ (mod 1021)
1000 = __ (mod 1021) (hint: think negative)
500 + 1000 = __ + __ = __ (mod 1021)

Multiplication

Working mod 6, calculate:

3 × 3 = __ (mod 6)
9 × 3 = __ (mod 6)
9 × 9 = __ (mod 6)

5 × 2 = __ (mod 6)
-1 × 2 = __ (mod 6)
-1 × -4 = __ (mod 6)

Inversion, “Division”

A ***multiplicative*** *inverse* of x is a number y such that x × y = 1. 0 has no multiplicative inverse.

Find the multiplicative inverse of each number, mod 5.

0	1	2	3	4

What is invertible, mod 6? If not invertible, cross out the box. If invertible, find the inverse.

0	1	2	3	4	5

What is invertible, mod 9? If not invertible, cross out the box. If invertible, find the inverse.

0	1	2	3	4	5	6	7	8

Modular Algebra

Use the inverses found above to solve:

4x = 1 (mod 5) 4x = 1 (mod 6) 4x = 1 (mod 9)

2x = 3 (mod 5) 5x = 4 (mod 6) 5x = 4 (mod 9)

Exponentiation

$x^n = x \cdot x \cdot \ ... \ \cdot x,$ n times.

In the world of modular arithmetic, it is not true that you can reduce the exponent, mod n. For example: $3^{10} \neq 3^3 (mod\ 7)$, even though $10 = 3\ (mod\ 7)$.

However, because of the properties of multiplication, you can reduce the base of the exponentiation: $10^3 = 3^3 (mod\ 7)$.

Calculate:

1^5 = __ (mod 7) 9^3 = __ (mod 7)

6^2 = __ (mod 7) 9^6 = __ (mod 7)

Find the remainder of: $13\overline{)99^7}$

1. Find the remainder of $13\overline{)99}$.
   99 = __ * 13 + __.
2. Therefore, 99 = __ (mod 13).
3. Raise that value to the power of 7.
   Don't make it too hard on yourself! Work in chunks: $x^7 = x^3 \times x^2 \times x^2$
4. Finally, we get that: 99^7 = ___ (mod 13).