Problems from Smullyan III[1]

For our last session of problems from Smullyan's book, we will solve a series of problems about two box-makers, Bellini and Cellini. Whenever Bellini fashions a box, he always writes a true inscription on it; and whenever Cellini fashions a box, he always writes a false inscription on it. Both Bellini and Cellini have sons who also make boxes, who follow the same inscription-writing habits as their fathers.

1. I once saw a box that had the following inscription:
    THIS BOX WAS NOT MADE BY ANY SON OF BELLINI.
    Who made it, Bellini, Cellini, a son of Bellini, or a son of Cellini?

2. Another time I saw a box whose inscription allowed me to conclude that the box must have been made by Cellini. Can you figure out what the inscription must have been?

3. The most valuable boxes of all are those bearing an inscription such that one can deduce that the box must have been made by Bellini or Cellini, but one cannot determine which one. I once had the good fortune to come across such a box. Can you figure out what the inscription must have been?

4. Suppose you came across a basket bearing the following inscription:
    THIS BOX WAS MADE BY ME.
    What would you conclude?

5. A Florentine nobleman gave lavish entertainments, the best of which was a game in which the prize was a valuable jewel. This nobleman had three boxes, gold, silver, and lead, and inside one of them was the jewel. He explained to the company that each of the boxes was made by Bellini or Cellini (and not any of their sons). The first person who could guess which box contained the jewel, and who could prove his guess correct, would be awarded the jewel.
    The inscriptions were:
    - Gold: IF THE JEWEL IS IN THE SILVER BOX THEN THE SILVER BOX WAS FASHIONED BY BELLINI
    - Silver: IF THE JEWEL IS IN THIS BOX THEN THE GOLD BOX WAS FASHIONED BY CELLINI
    - Lead: THE BOX WHICH REALLY CONTAINS THE JEWEL WAS FASHIONED BY CELLINI

    Which box contained the jewel?

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[1] These problems have been liberally adapted from Smullyan's *What is the Name of this Book?*, which possesses more interesting logic puzzles like these than I could conceivably borrow.

A slight change of rules. In some museums can be found pairs of boxes—one gold, one silver—made and sold as sets. Actually, Bellini's and Cellini's families have been friends for generations and would sometimes collaborate in making a pair, so that sometimes one person would make one box and another person would make a second. They delighted in writing their inscriptions so that someone could figure out who had been the makers.

1. I once came across the following pair:
    - Gold: BOTH BOXES OF THIS SET WERE MADE BY MEMBERS OF THE CELLINI FAMILY
    - Silver: NEITHER OF THESE BOXES WAS MADE BY ANY SON OF BELLINI OR ANY SON OF CELLINI.

    Who made each box?

2. I once came across the following pair:
    - Gold: IF THIS BOX WAS MADE BY ANY MEMBER OF THE BELLINI FAMILY THEN THE SILVER BOX WAS MADE BY CELLINI
    - Silver: THE GOLD BOX WAS MADE BY A SON OF BELLINI

    Who made each box?

3. Consider this pair:
    - Gold: THE SILVER BOX WAS MADE BY A SON OF BELLINI
    - Silver: THE GOLD BOX WAS NOT MADE BY A SON OF BELLINI

    Prove that at least one of them was made by Bellini.

4. Consider the following pair:
    - Gold: THE SILVER BOX WAS MADE BY CELLINI
    - Silver: THE GOLD BOX WAS NOT MADE BY CELLINI

    Prove that at least one of the boxes was made by a son of Cellini.

5. Consider the following pair:
    - Gold: THE SILVER BOX WAS MADE BY A SON OF BELLINI
    - Silver: THE GOLD BOX WAS MADE BY A SON OF CELLINI

    Prove that at least one of the boxes was made by Bellini or Cellini.

HW: How many different combinations of Bellini, Cellini, sons of Bellini, or sons of Cellini, could have made the boxes?