

# Pile Splitting Games III

BMC Intermediate I Spring 2023

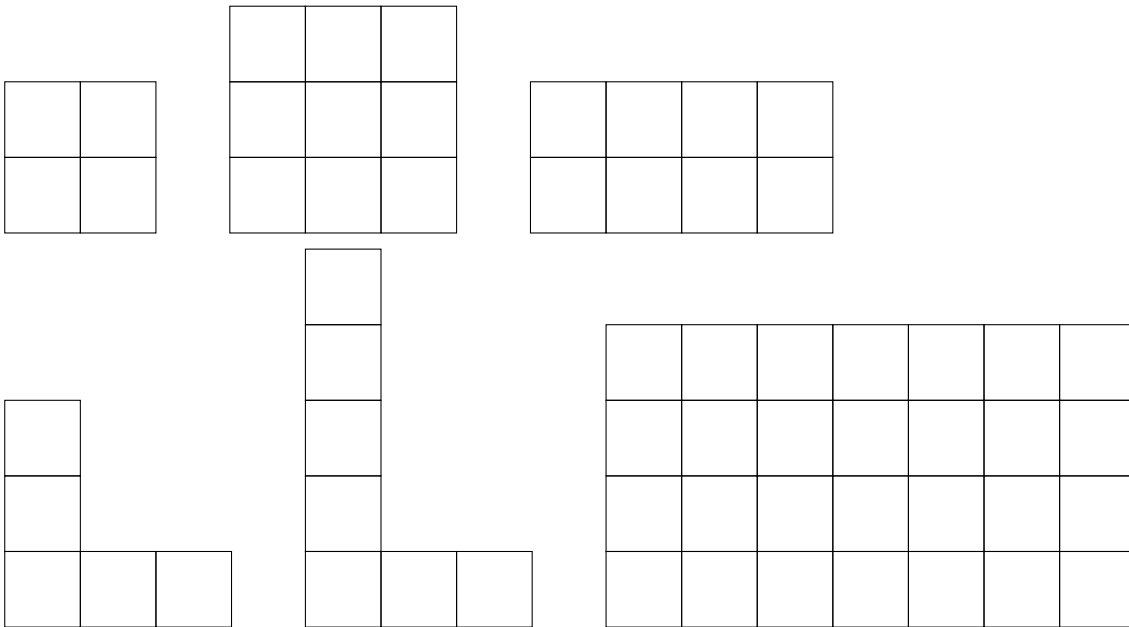
April 19, 2023

## 1 Chomp

Chomp is a game that is played with a grid of squares. Each turn, a player can pick a square and take it along with all the squares that are above and to the right of it. The player who takes the bottom left square loses.



1. Play Chomp on the following rectangular grids. For which boards is it better to go first or second?

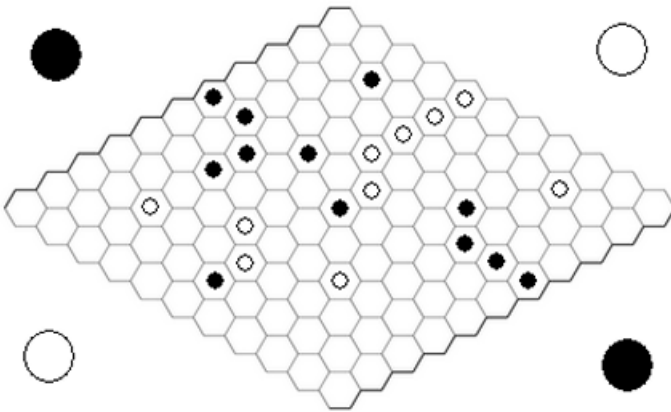


2. What should be your strategy if you go first when the chocolate bar is an  $n \times n$  square?
3. What is a winning strategy if the chocolate bar is a  $2 \times n$  board?

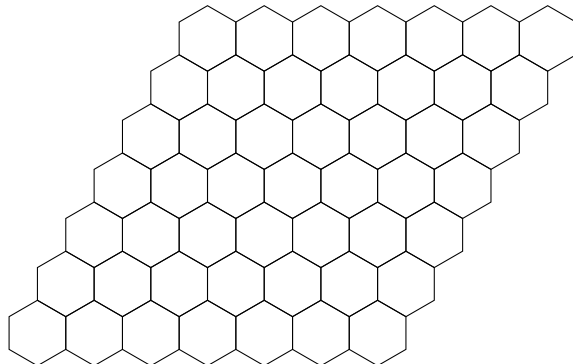
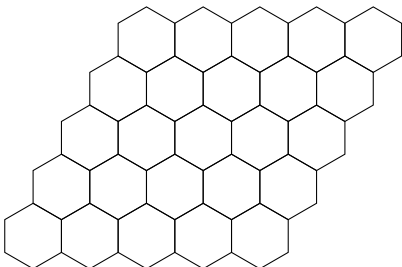
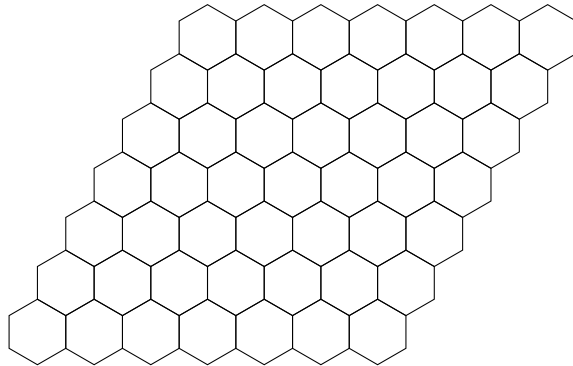
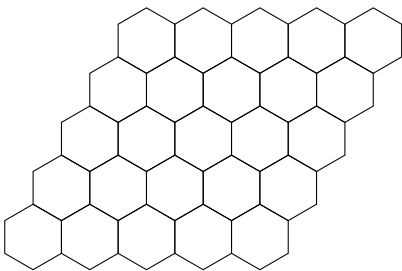
- Does the second player always lose for a rectangular grid (with more than one square)? Hint: If there was a grid where the second player could win, then there is a way for them to win if the first player removed the square at the top right corner. Can we get a contradiction?

## 2 Hex

Hex is played with two people on a hexagonal grid. Two opposite sides are labeled black or X, and two are labeled white or O. Players take turn placing X's and O's in empty spaces in the grid. The goal for each player is to form an unbroken chain of hexagons of their shape linking their two sides.



- Play Hex on these small boards with a friend. (Repeated for multiple playthroughs)



2. Is it possible for this game to end in a draw?
3. Does the first or second player have a winning strategy? Hint: Think in terms of strategy stealing like in Chomp. If the second player has a winning strategy and the first player plays randomly, can the first player then steal the second player's strategy?
4. What if you give the second player an advantage by giving them a shorter distance to cover. Does the first player always win still?

