Pile Splitting Games II

BMC Intermediate I Spring 2023

April 12, 2023

1 Pile Splitting

1. We now consider two player games involving splitting piles of stones. We start with 21 stones and each player may take $1, 2$, or 3 stones from the pile. The first person who cannot take any stones from the pile loses. Can the either player guarantee victory and how?

2. If the game changes to the first person who can't make a move wins, how does that change this game?

3. Let's suppose that you can take any power of two number of stones from the pile. So you could take 1, 2, 4, 8, 16, ... stones. For which starting number of stones is the first player guaranteed to win?

4. (Challenge) Suppose that you can take any prime number of stones from the pile. Prove that there are infinitely many number of starting stones that are losing positions.

5. Now suppose there are two piles of stones and on each turn, you can take $1, 2$, or 3 stones from a single pile. If the pile sizes are $(4, 4)$, who wins? What about $(4, 5)$? $(4, 6)$?

6. Consider a game with two piles of stones. Each turn, a player takes away one pile of stones and splits the other pile into two smaller piles. The first player who cannot split the remaining pile loses. Can either player guarantee victory and how?

7. (Challenge) Consider another game consisting of 5 squares. At each step, a player can either place a stone in an empty square or they can remove a stone from a square and place a stone in the nearest empty square to the left and to the right. (If no empty square is available, then no stone is placed). The game ends when there is only one empty square. The last player to make a move wins. What is the winning strategy? What if there are 7 squares? Any odd number of squares?