BMC - Advanced

CATCHING THE BUTTERFLY

with Harry

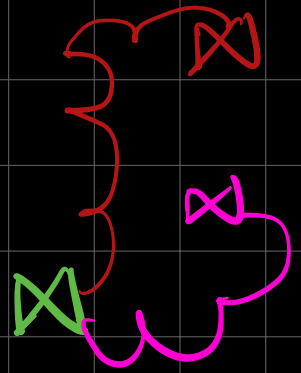
(1) Set-up

Infinite chess board

Players → Butterfly ⋈ of power n

each turn, the butterfly may move any amount $\leq n$ squares.

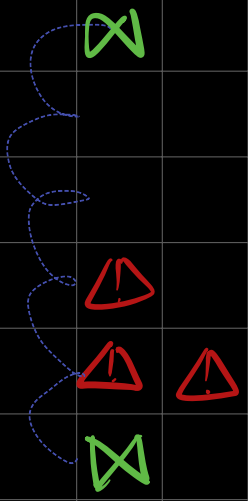
Eg 5-butterfly: Cannot land on a trap.



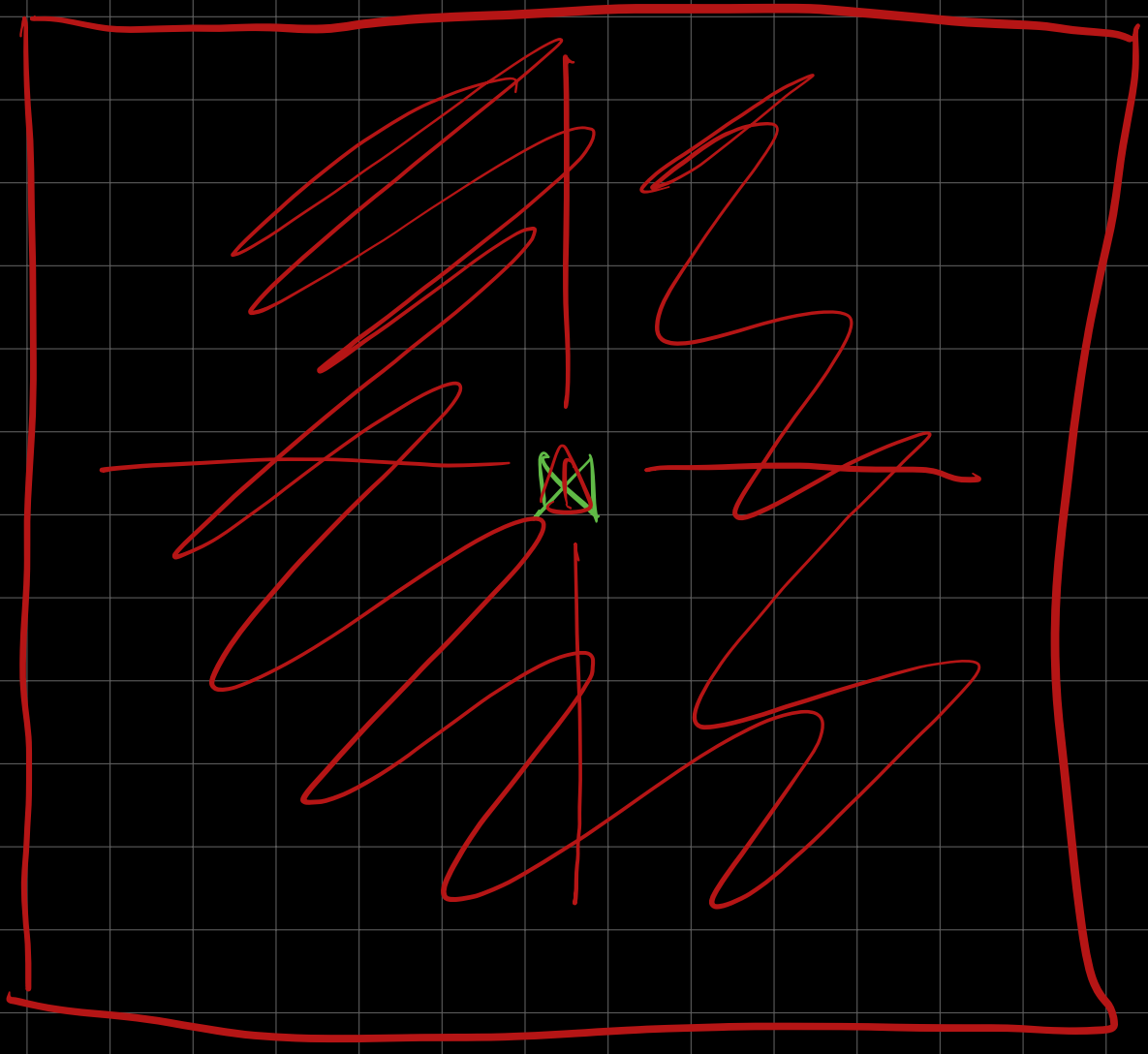
→ Catcher

each turn, catcher may set a trap in any square

Eg:



→ The butterfly wins if it can always move.
→ The catcher wins if the traps are set up so that the butterfly is trapped.



Q1: Who has the advantage?

The number of moves are exponentially more "complex" for the catcher compared to the butterfly.

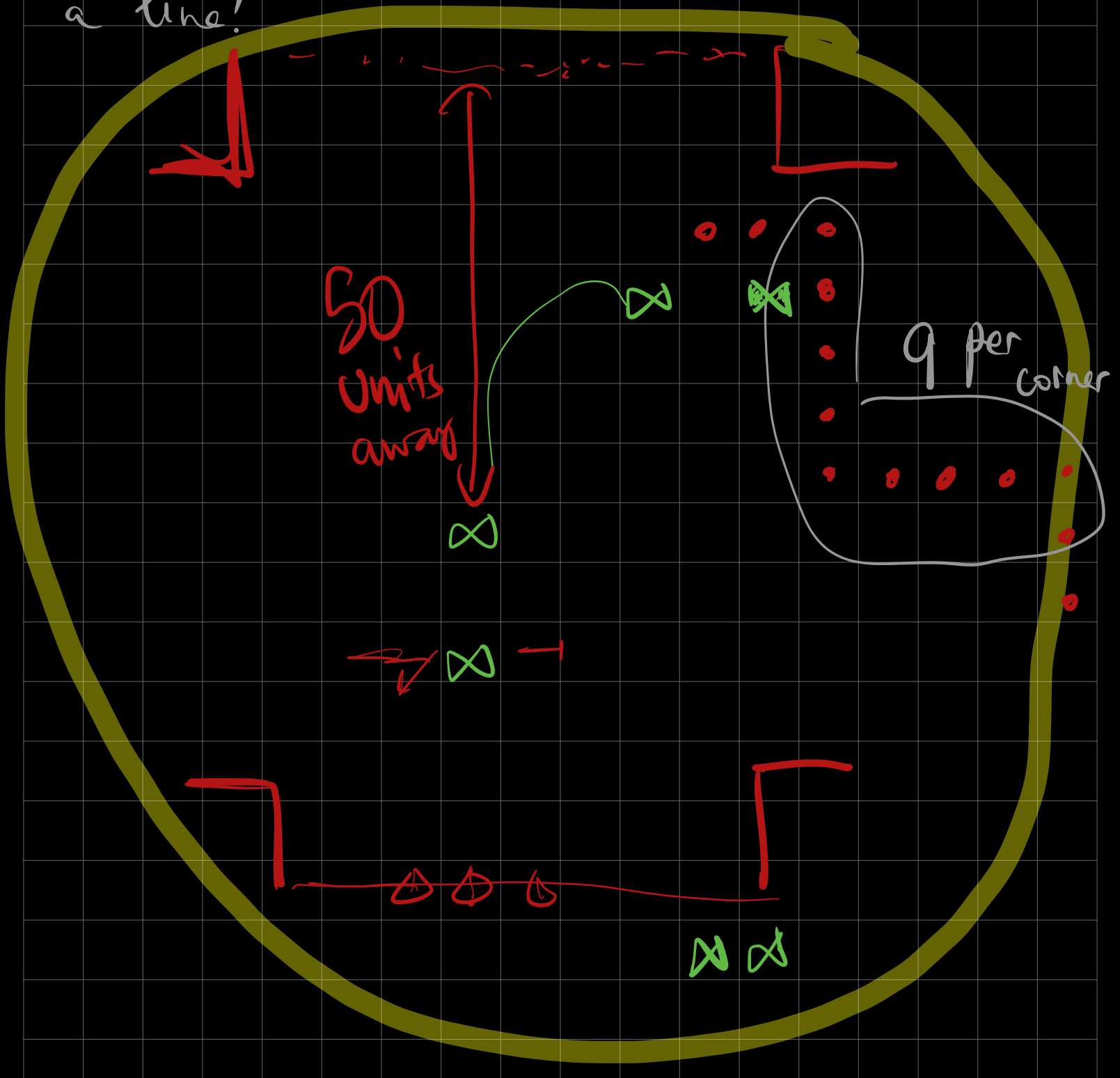
The catcher NEVER makes a mistake!

Play 1- butterfly vs catcher.

Q2: Is this game determined?
(Does there exist a winning strategy for this game?)

How many turns does it take to finish the square before the butterfly reaches the wall?

If butterfly is below, how many traps does it take to force the butterfly to stay below a line?



Strategy #2:

Q3 Can we trap 2-butterfly?

(Break until 7 PM)

This is a $1000 question.

Q4 How do we know that a winning strategy exist?

Lemma A winning strategy MUST exist for this game.

Pf: Case 1: The catcher has a winning strategy → done

Case 2: The catcher does NOT have a winning strategy.

∞ plays as follows:

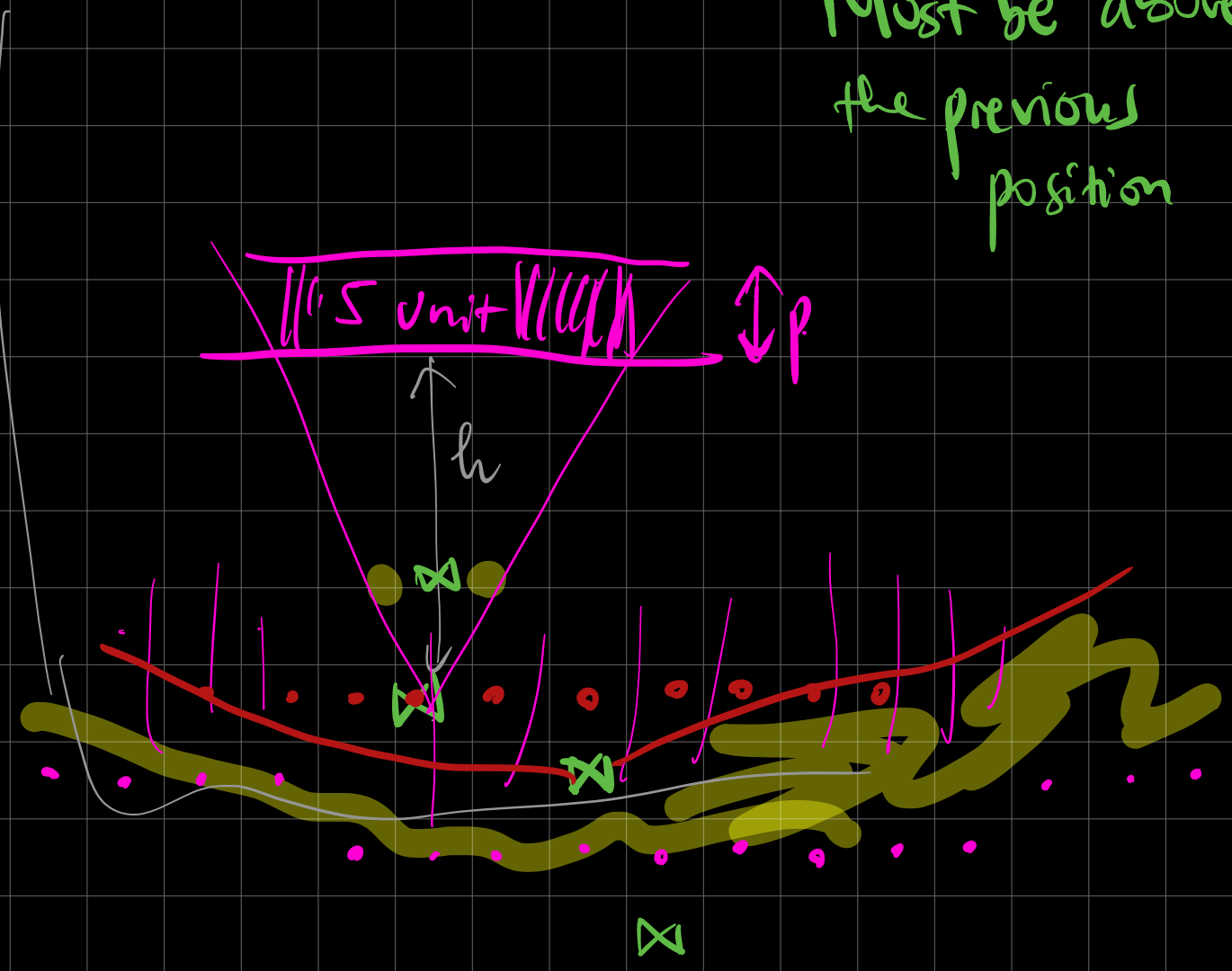
In each turn, go where the result is that the catcher can't win.

→ Look at a "simpler" problem.

Catching the Moth

* Moth = Butterfly that always flies towards the sun ie. ending position

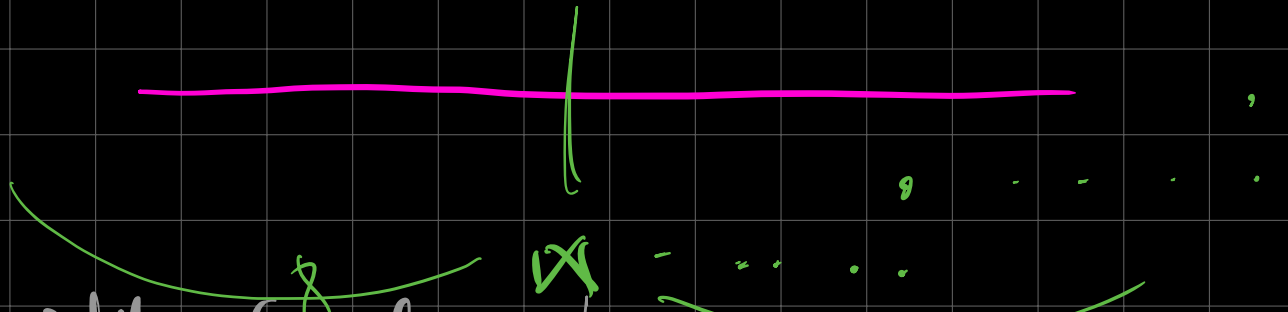
Must be "above" the previous position



Where can the moth end up?

If ⊗ has power 5, and the wall is h units away, how wide is the wall?

$$(2h(p-1)+1)\cdot p$$



h	width of wall, w
1	9
2	17
h	$8\cdot h+1$
	$2(p-1)h+1$

Q: If we start building a wall p-unit thick, h distance away from the moth,
↳ very large

will we have enough time to finish a relevant block of the wall that caps the cone of the moth?