

Beginner: Invariant Games II

BMC Beg Spring 2023

March 22, 2023

1 Pile Splitting

1. We now consider two player games involving splitting piles of stones. We start with 21 stones and each player may take 1, 2, or 3 stones from the pile. The first person who cannot take any stones from the pile loses. Can the either player guarantee victory and how?
2. Consider a game with two piles of stones. Each turn, a player takes away one pile of stones and splits the other pile into two smaller piles. The first player who cannot split the remaining pile loses. Can either player guarantee victory and how?
3. Now consider a game with two piles of stones where on each turn, a player can take however many stones they want from a single pile. The first player who can't make a move loses. Can either player guarantee victory and how?
4. What if we consider the previous game with three piles of stones instead of two? When can the first player win and how?