

Beginner: Invariant Games

BMC Beg Spring 2023

March 15, 2023

1 Pile Splitting

1. Take a pile of 10 stones. Each turn, you can take a pile of at least two stones and then split it into two piles. Write down the product of the two smaller piles you just made. Keep splitting piles until you cannot anymore and then add up all the products. What is the largest sum you can get? What is the smallest sum you can get?
2. How many splits do you do? How many of those splits are odd-odd?
3. Instead of writing the product $a \cdot b$, write $a \cdot b \cdot (a + b)$. What is the total sum you get at the end?
4. Instead of writing the product $a \cdot b$, write the product of a 1's and b 1's. So if $a = b = 2$, we would write $11 \cdot 11 = 121$. What is the total sum you get at the end?
5. Instead of writing the product, write $\frac{1}{a} + \frac{1}{b}$ and then multiply all the products together. What is the largest product you can make? What is the smallest product you can make?