Combinatorial Games II

BMC Beginner I Fall 2023

October 25, 2023

1 Pile Splitting Game

1. We start with 21 stones and each player may take $1, 2$, or 3 stones from the pile. The first person who cannot take any stones from the pile loses. Can the either player guarantee victory and how?

2. Now suppose that each player may take either $2, 5$, or 6 stones from the pile. Suppose that if there is one stone left, then the game ends in a tie. If there are no stones left, then the person who took stones last won. Draw a tree of the different possibilities.

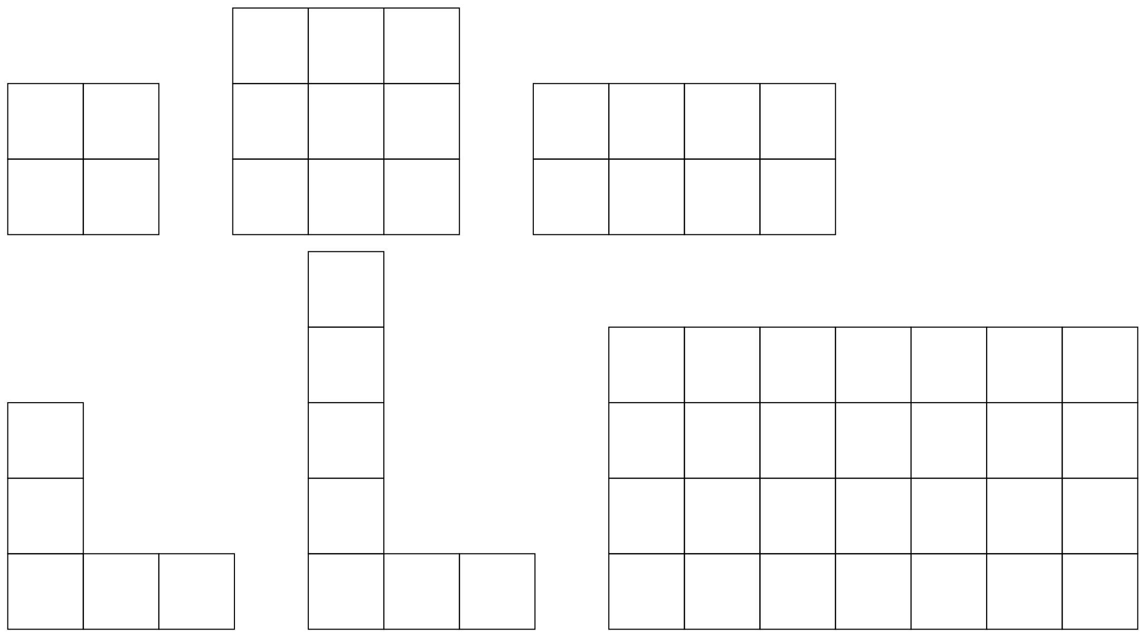
3. Label every node in the game tree by who can win in that state.

   Theorem 1.1 (Zermelo's Theorem)**.** In all combinatorial games, exactly one player has a winning strategy, or both players have a draw strategy.

2 Chomp

Chomp is a game that is played with a grid of squares. Each turn, a player can pick a square and take it along with all the squares that are above and to the right of it. The player who takes the bottom left square loses.

1. Play Chomp on the following rectangular grids. For which boards is it better to go first or second?



2. What should be your strategy if you go first when the chocolate bar is an $n \times n$ square?

3. What is a winning strategy if the chocolate bar is a $2 \times n$ board?

4. Does the second player always lose for a rectangular grid (with more than one square)? Hint: If there was a grid where the second player could win, then there is a way for them to win if the first player removed the square at the top right corner. Can we get a contradiction?