Exercises on p-adic numbers. I

1. What number is represented by $\ldots 31313131$ in 10-adic numbers? And in 5-adic?

2. Express the numbers $1/7, 2/7, 3/7, 4/7, 5/7, 6/7$ as "usual" periodic decimal fractions. Now represent them as 10-adic numbers. Do you notice anything interesting?

3. We know that the equation $x^2 = x$ has 4 solutions in 10-adic numbers:

$$0, \quad 1, \quad \ldots 39225625991821289 0625 \quad \text{and} \quad \ldots 60774374008178710 9376.$$

   How many solutions does $x^3 = x$ have in 10-adic numbers?

4. We saw that 10-adic numbers have *zero divisors*, i.e. non-zero numbers whose product is zero. Show that if p is prime, then p-adic numbers do not have zero divisors.

5. (a) We saw that there is no $\sqrt{2}$ in p-adic numbers for $p = 2, 3$, or 5. Show that it exists for $p = 7$.
   (b) One of the values of $\sqrt{2}$ in 7-adic numbers ends with digits $\ldots 13$. Find the next three digits.
   (c) Show that the digits of $\sqrt{2}$ in 7-adic numbers are not periodic or eventually periodic.