BMC Beginners II: Modular Arithmetic

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Definition: For positive integer $m$, $a$ and $b$ are congruent modulo $m$ if $a$ and $b$ have the same remainder when divided by $m$.

$$a \equiv b \mod m$$

Use the notation for modular arithmetic when solving the following problems.

1. If we have a group of 38 people and we want to make groups of 3, how many people are not in a group?

2. Today is a Wednesday. What day of the week is it 30 days from now?

3. Your neighbor starts building their fence on Monday morning. They work only on weekdays (Mon-Fri), and it takes them 32 days to build the fence. On what day does your neighbor finish building the fence?

4. On 9pm on Tuesday, PG&E had a planned power outage for 100 hours. What day and time does the electricity come back?

5. Suppose Sally is on a snowboarding expedition on the moon. She wants to break the world record and complete a $3150^\circ$ rotation. If she starts facing due west and turns counter-clockwise, what direction should she be facing at the end?

6. A gaggle of turkeys live on Farmer John’s farm. Farmer John sends his 3 kids out to count the number of turkeys. The 3 kids, Alice, Bob, and Charles, come up with a plan. Alice will count by 3’s, Bob will count by 5’s, and Charles will count by 19’s. After counting the turkeys, they report back to Farmer John.

   Alice says: “I counted the turkeys by 3’s and there were 2 left over.”
   Bob says: “I counted the turkeys by 5’s and there were 3 left over.”
   Charles says: “I counted the turkeys by 19’s and there were 7 left over.”

   What is the minimum number of turkeys that Farmer John can have?
Properties: Suppose $a_1 \equiv b_1 \mod m$ and $a_2 \equiv b_2 \mod m$. Then, we have the following.

- $a_1 + a_2 \equiv b_1 + b_2 \mod m$
- $a_1 \cdot a_2 \equiv b_1 \cdot b_2 \mod m$

Use these properties when solving the following problems.

7. Find a divisibility rule for 8. Make sure to explain why it works.

8. Testing for divisibility for 24 using 4 and 6 doesn’t work. Can you find a way that does work?

9. Let $n$ be a natural number. $n^2$ has a remainder of 4 when divided by 5 and $n^3$ has a remainder of 2 when divided by 5. What is the remainder of $n$ when divided by 5?