Notation

- The *derivative* of $y = f(x)$ is denoted as $f'(x)$, y', $\frac{df}{dx}$, $\frac{dy}{dx}$, or $\frac{d}{dx}(f(x))$. These are all the same!
- The derivative of $y = f(x)$ evaluated at a point $x = a$ is denoted as $f'(a)$, $\frac{dy}{dx}|_{x=a}$, or $\frac{dy}{dx}|_{x=a}$

Derivative Properties and Formulas

Suppose $g(x)$ and $f(x)$ are differentiable functions (meaning their derivatives exist), and $c \in \mathbb{R}$.

- $\frac{d}{dx}(c) = (c)' = 0$ - derivative of a constant is 0
- $(cf(x))' = cf'(x)$ and $(f(x) + g(x))' = f'(x) + g'(x)$ - derivative behaves “linearly”
- $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$ - Product Rule
- $(\frac{f(x)}{g(x)})' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ - Quotient Rule
- $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$ OR $\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}$ - Chain Rule

Useful Derivatives

- For $n \in \mathbb{R}$, $\frac{d}{dx}(x^n) = nx^{n-1}$ - Power Rule
- $\frac{d}{dx}(\sin(x)) = \cos(x)$
- $\frac{d}{dx}(\cos(x)) = -\sin(x)$
- $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$ for $x > 0$
- $\frac{d}{dx}\frac{1}{f(x)} = \frac{-f'(x)}{(f(x))^2}$
- $\frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x)$

Special Functions and their Derivatives

- Sigmoid function - $\sigma(x) = \frac{1}{1+e^{-x}}$ and $\sigma'(x) = \sigma(x)(1 - \sigma(x))$
- Rectified Linear Unit - $\text{ReLU}(x) = \max(0, x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$ and $\frac{d}{dx}(\text{ReLU}(x)) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$
- Hyperbolic Sin - $\sinh(x) = \frac{e^x - e^{-x}}{2}$ and $\sinh'(x) = \cosh(x)$
- Hyperbolic Cos - $\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\cosh'(x) = \sinh(x)$
- Hyperbolic Tan - $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ and $\tanh'(x) = \frac{1}{\cosh^2(x)} = \text{sech}^2(x)$