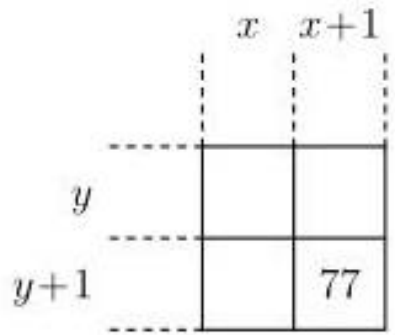
BERKELEY MATH CIRCLE

Math Kangaroo II:
Continued Explorations
in
The Art of Problem Solving

Instructor: Patricio Angulo via Oye Productions

A square of numbers is taken out from a multiplication table. Only one number is visible. The integers x and y are both positive and x is greater than y. What is the value of x?



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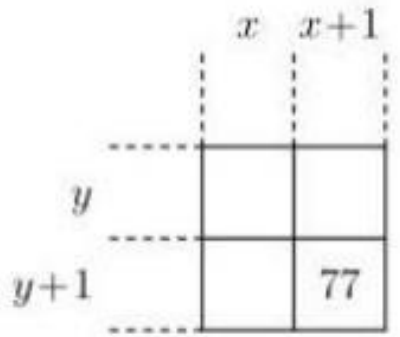


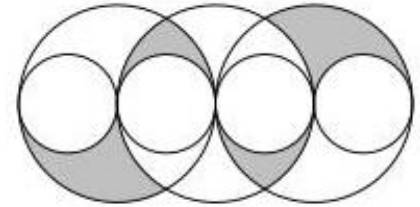
$(x+1)(y+1) = 77$ factor

$77 = 7 \times 11$

$7 = y+1 \Rightarrow y = 6$

$11 = x+1 \Rightarrow x = 10$

The diagram shows three large circles of equal radius and four small circles of equal radius where the centers of all circles and all points of contact lie on one straight line. The radius of each small circle is 1. What is the shaded area?



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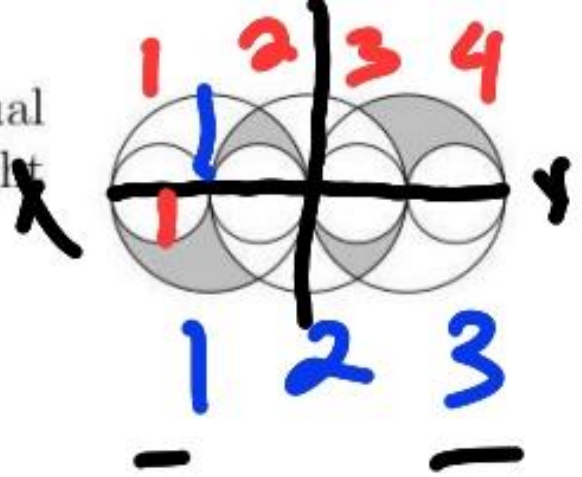

B: 2

$S = \pi r^2 - \pi \cdot 1^2 = \pi$

$B = \pi \cdot 2^2 = 4\pi$

$\text{Area} = 4\pi - \pi - \pi = 2\pi$

What is the greatest common divisor of $2^{2021} + 2^{2022}$ and $3^{2021} + 3^{2022}$?

$2^{2021}(1+2)$

$(2^{2021})(3)$

$(3^{2021})(1+3)$

$(3^{2021})(2^2)$

What's common?

$3 \cdot 2^2 = \boxed{12}$

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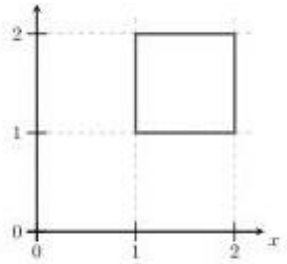
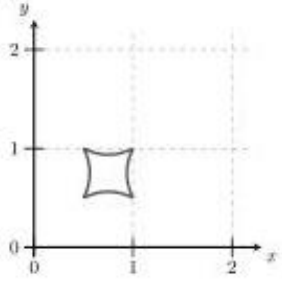
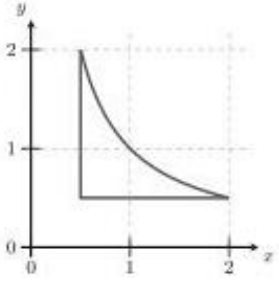
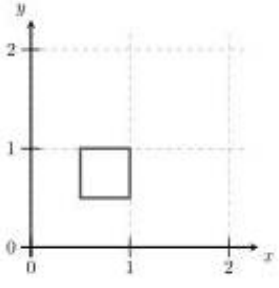
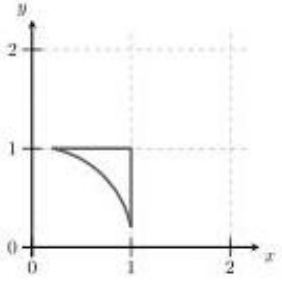
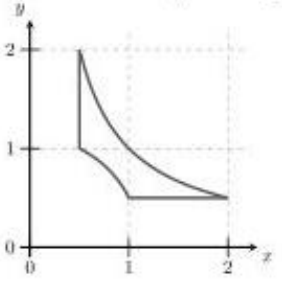


(A) 

(B) 

(C) 

(D) 

(E) 

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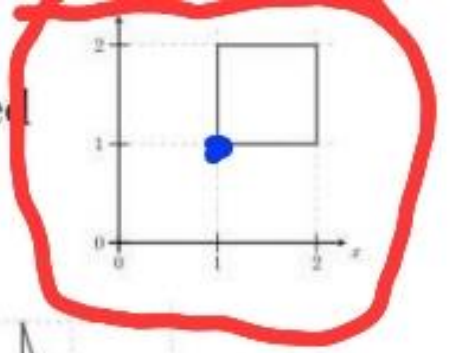
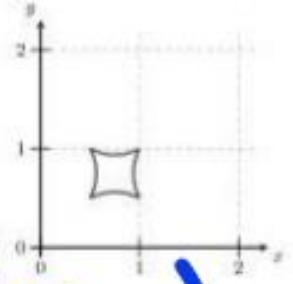
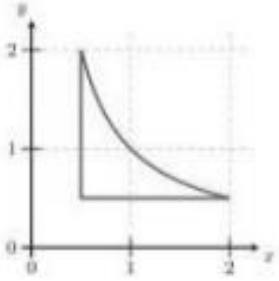
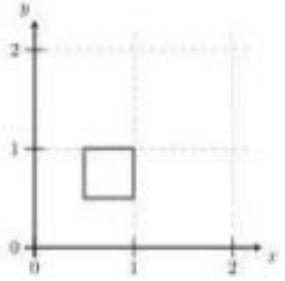
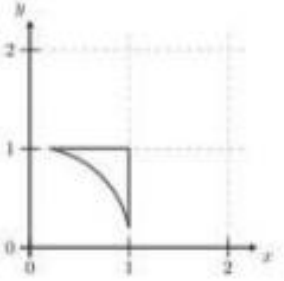
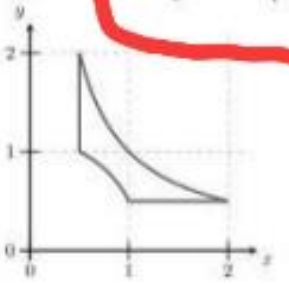


(A) 

(B) 

(C) 

(D) 

(E) 

B(1,2) C(2,2) A D(2,1) (1,1)

$\rightarrow$ A(1,1) B(1, $\frac{1}{2}$) C($\frac{1}{2}$, $\frac{1}{2}$) D($\frac{1}{2}$, 1)

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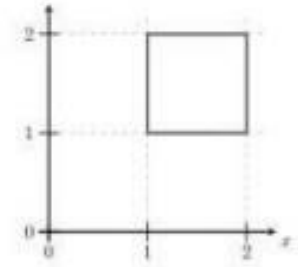
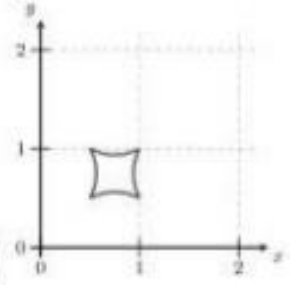
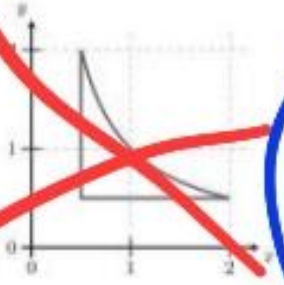
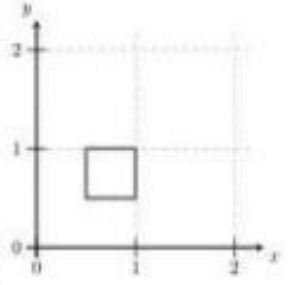
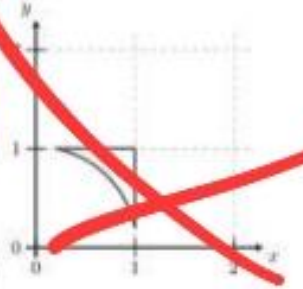
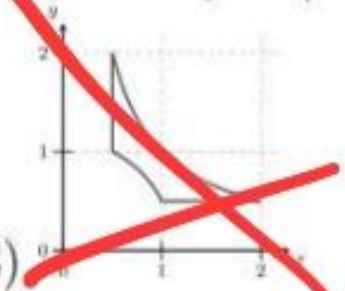


(A) 

(B) 

(C) 

(D) 

(E) 

$\left(\frac{1}{2}, 1\right)$ $(1,1)$

$(2,1)$ $(1,1)$

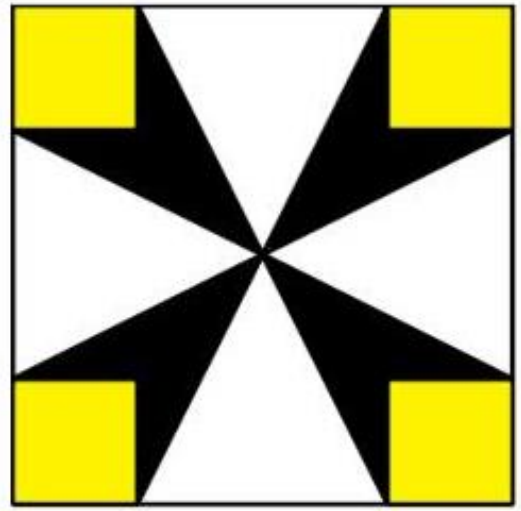
$\left(\frac{1}{3}, 1\right)$ $(1,1)$ $\left(\frac{1}{2}, \frac{1}{2}\right)$ $\left(1, \frac{1}{3}\right)$

$\left(\frac{1}{2}, 1\right) \rightarrow (1,1)$

Y

Y = Straight

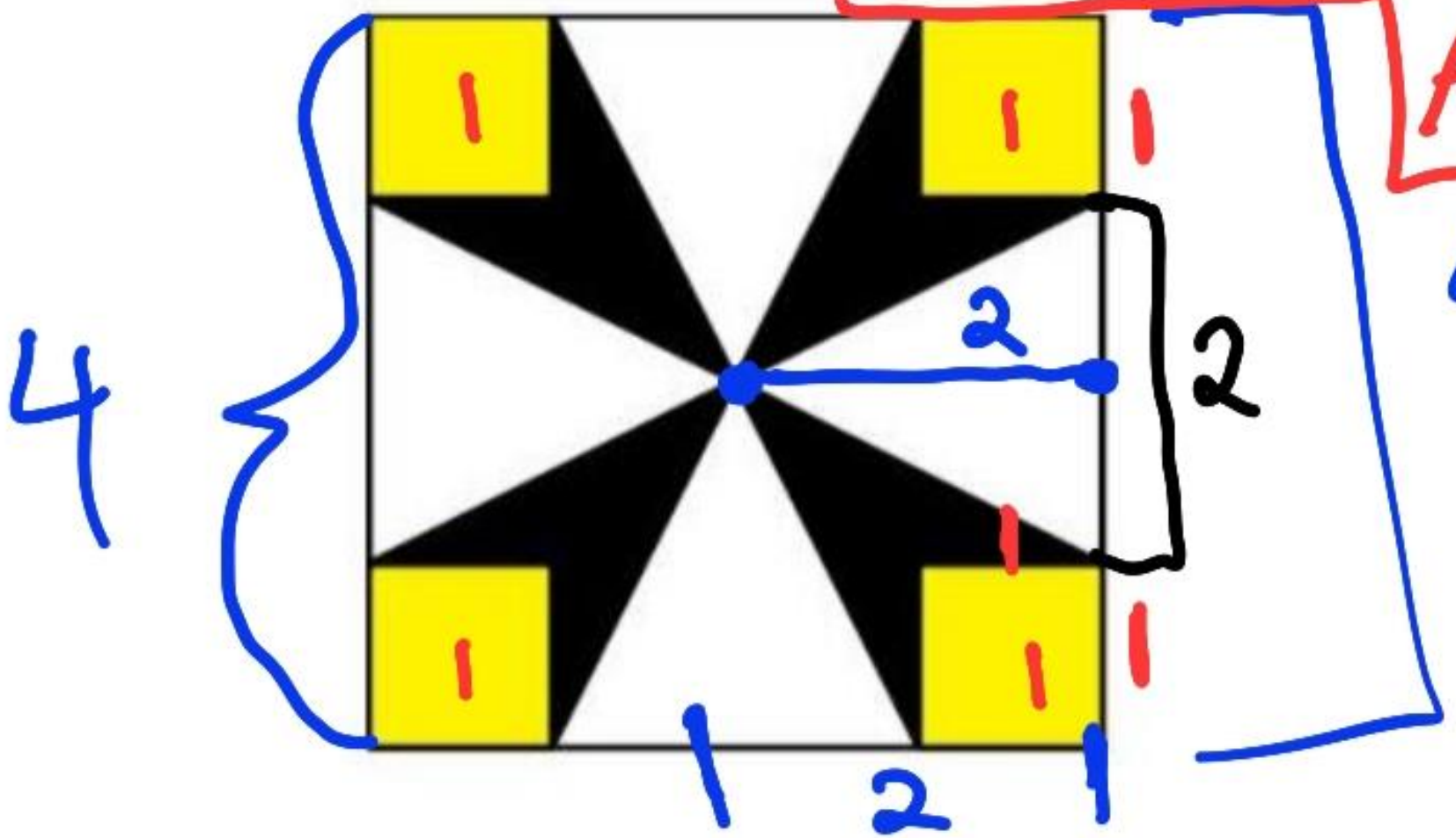
13. The area of the large square is 16 cm^2 and the area of each small square is 1 cm^2. What is the total area of the black flower?



Square = 16 $\Rightarrow$ 4×4

$A = 16 - 4(1) - 4(2)$

$A = 4\ cm^2$



Area $\Delta = \frac{1}{2}bh = (\frac{1}{2})(2)(2) = 2$