

BERKELEY MATH CIRCLE

The Math of Chemistry:

**Chemical Reactions
&
Equilibrium**

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The Periodic Table

Periodic Table of the Elements

1 IA 11A																	18 VIIIA 8A
1 H Hydrogen 1.008																	2 He Helium 4.003
3 Li Lithium 6.941	4 Be Beryllium 9.012											5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180
11 Na Sodium 22.990	12 Mg Magnesium 24.305	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8 VIII 8	9 VIII 8	10 VIII 8	11 IB 1B	12 IIB 2B	13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.88	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.933	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.39	31 Ga Gallium 69.723	32 Ge Germanium 72.61	33 As Arsenic 74.922	34 Se Selenium 78.09	35 Br Bromine 79.904	36 Kr Krypton 84.80
37 Rb Rubidium 84.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.94	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.905	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.71	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.904	54 Xe Xenon 131.29
55 Cs Cesium 132.905	56 Ba Barium 137.327	57-71 Lanthanide Series	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.85	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium [208.982]	85 At Astatine 209.987	86 Rn Radon 222.018
87 Fr Francium 223.020	88 Ra Radium 226.025	89-103 Actinide Series	104 Rf Rutherfordium [261]	105 Db Dubnium [262]	106 Sg Seaborgium [266]	107 Bh Bohrium [264]	108 Hs Hassium [269]	109 Mt Meitnerium [268]	110 Ds Darmstadtium [269]	111 Rg Roentgenium [272]	112 Cn Copernicium [277]	113 Uut Ununtrium unknown	114 F1 Flerovium [289]	115 Uup Ununpentium unknown	116 Lv Livermorium [296]	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown

57 La Lanthanum 138.906	58 Ce Cerium 140.115	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.24	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.966	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.50	67 Ho Holmium 164.930	68 Er Erbium 167.26	69 Tm Thulium 168.934	70 Yb Ytterbium 173.04	71 Lu Lutetium 174.967
89 Ac Actinium 227.028	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.080	99 Es Einsteinium [254]	100 Fm Fermium 257.095	101 Md Mendelevium 258.1	102 No Nobelium 259.101	103 Lr Lawrencium [262]

Alkali Metal	Alkaline Earth	Transition Metal	Semimetal	Nonmetal	Basic Metal	Halogen	Noble Gas	Lanthanide	Actinide
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Chemical Reactions & Equilibrium

In Chemistry, we like to observe reactions as they happen. The reaction is considered to be done once it reaches what we call **Equilibrium**. At Equilibrium, the macroscopic reaction is done, but we do see things occurring on the microscopic level. For example,

Macroscopic



+

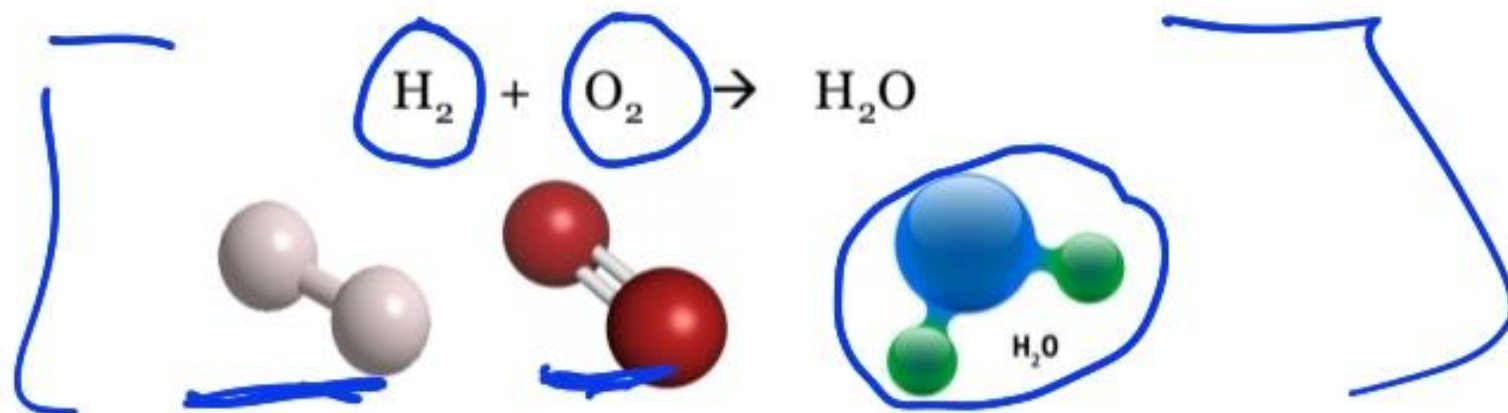


=



H₂O

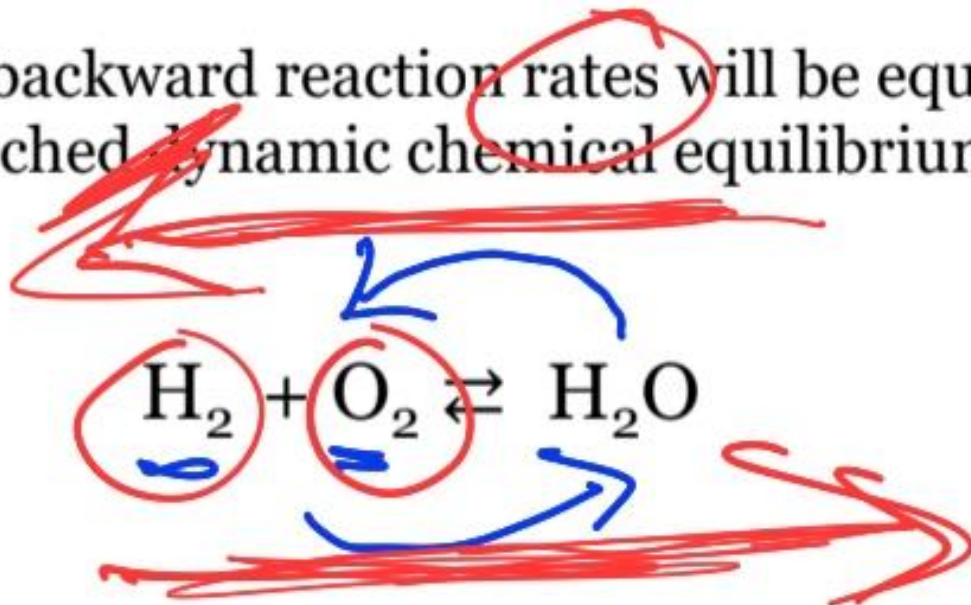
Microscopic



Chemical Reactions & Equilibrium

The reaction should continue until the reactants have been completely used up. However, this is not the case for most of the reactions because we have a **reverse reaction** taking place. Let us suppose that we begin our reaction by mixing two known reactants. At the beginning of the reaction, we have all the reactants and no products. As the reaction proceeds forward, the concentration of the reactants decreases. At the same time, the concentration of product begins to increase BUT some of the products begin to convert back to reactants.

Eventually the forward and backward reaction rates will be equal, and at this point we say that our reaction has reached dynamic chemical equilibrium.



Chemical Reactions & Equilibrium

We have a mathematical way to discuss this relationship, and that is referred to as the **Equilibrium Constant, K** (it's a fraction in essence, see below). The higher the equilibrium constant, the more products we have formed and the less reactants we have remaining. On the contrary, the smaller the equilibrium constant, the less product-favored our reaction, and the less product we have formed.



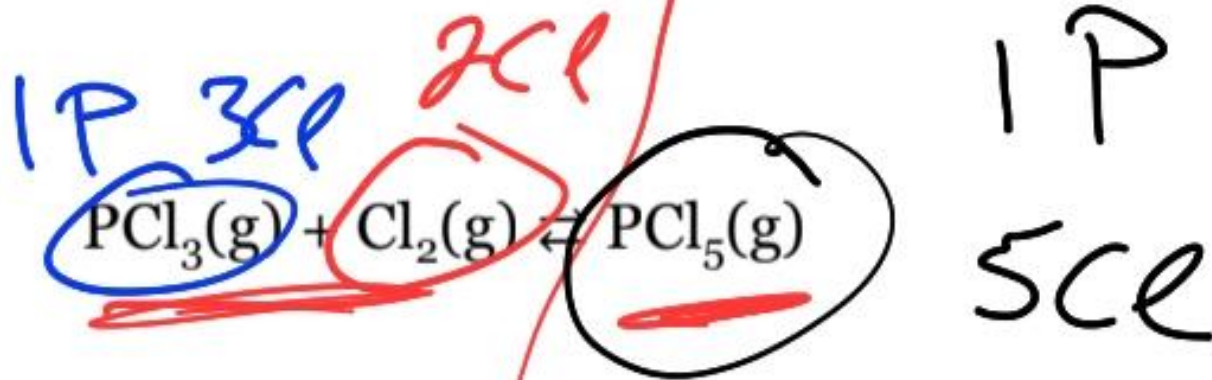
To do some math with this concept, we need to learn some fun background science-y things. Up first is learning how to balance chemical reactions. Here goes!

Chemical Reactions & Equilibrium

Balancing Chemical Reactions

The basic premise is that what goes in must come out, i.e., everything seen on the reactant side **MUST** also be seen on the product side. The forms of the molecules may have changed, but all the same atoms and the number of those atoms must be equal on both sides of the arrow. It's literally like an algebra equation!

For example:

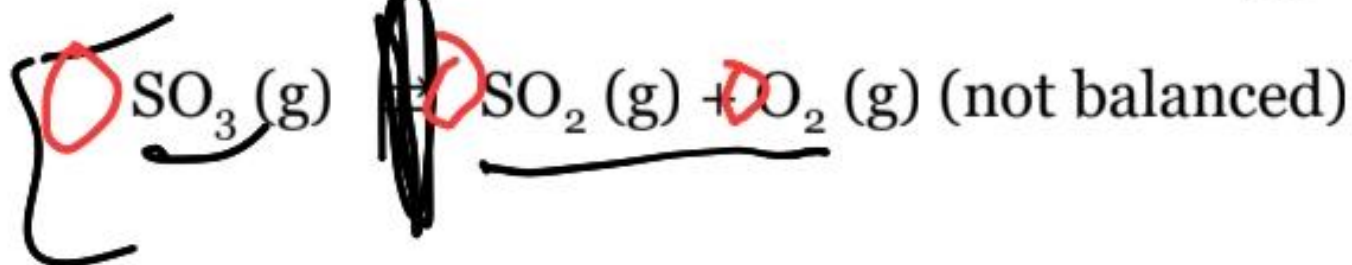


We observe on the reactant side, that we have one P and five Cl's.

We see the same on the product side.

Chemical Reactions & Equilibrium

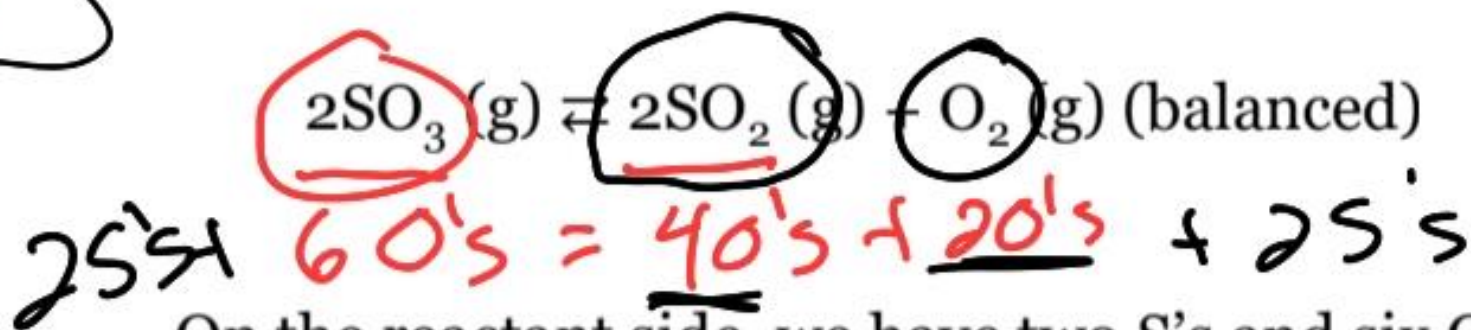
Balancing Chemical Reactions



On the reactant side, we have one S and three O's.

On the product side, we have one S and four O's.

We are not balanced!



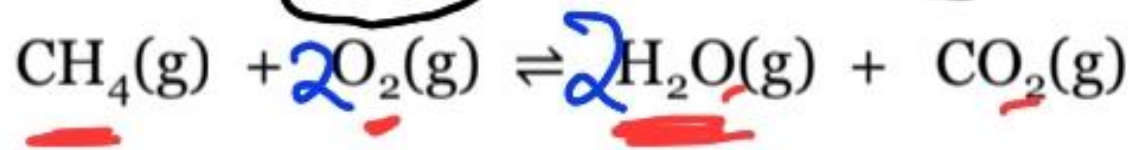
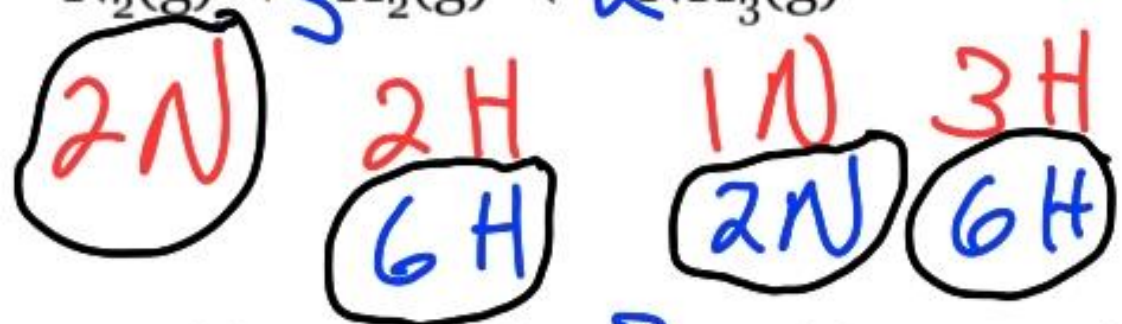
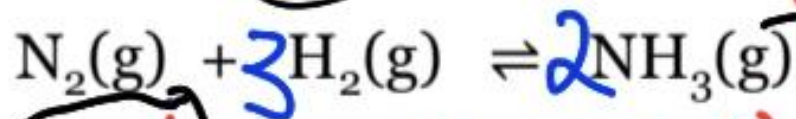
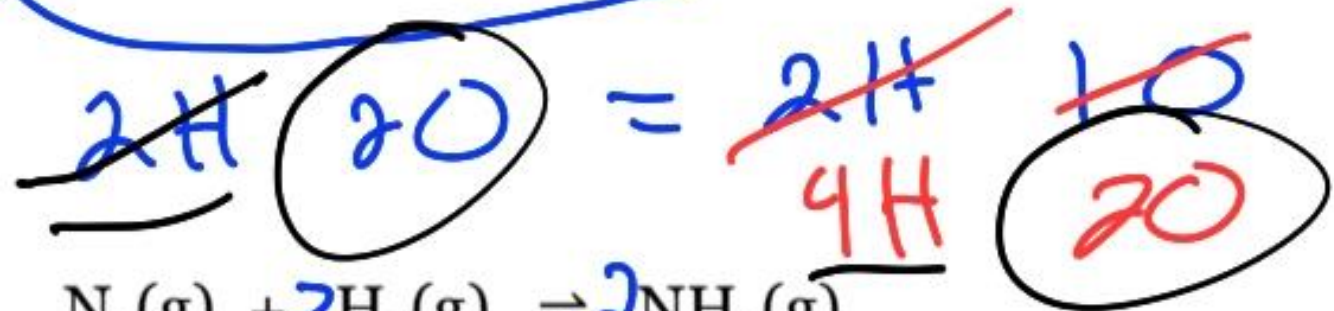
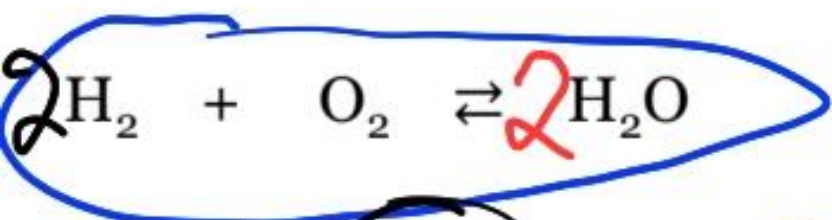
On the reactant side, we have two S's and six O's.

We now see the same on the product side.

Chemical Reactions & Equilibrium

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Let's Practice Balancing Chemical Reactions!



2 O's

4 O's



H, C,
Oxygen

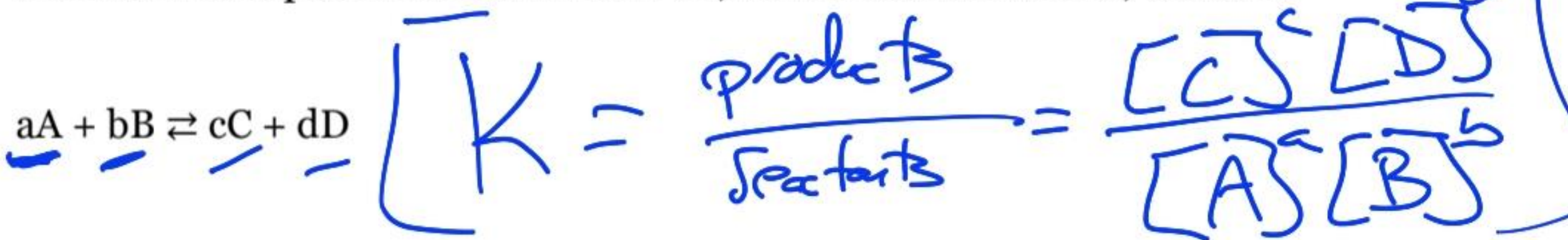
Chemical Reactions & Equilibrium

Now that we know how to balance chemical equations, we can set-up our:

Equilibrium Constant (K)

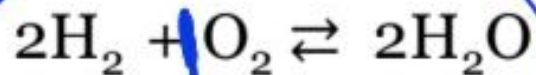
Chemical reactions can be characterized by an **equilibrium constant**, K . This constant expresses the ratio of the product of the reaction ~~products~~ to the product of the reactants.

We use the concentrations (or pressures) of each molecule (those values are given or figured out per the word problem). The coefficient from the balanced chemical becomes the exponent for each term. So, for the reaction below, we have:

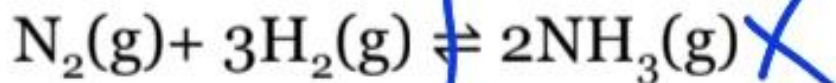


Chemical Reactions & Equilibrium

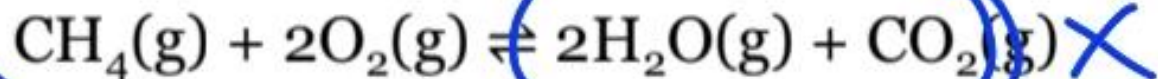
From our previous reactions, we would have:



$$K = \frac{\text{prod}}{\text{react}} = \frac{[\text{H}_2\text{O}]^2}{[\text{H}_2]^2 [\text{O}_2]}$$



$$K = \frac{[\text{NH}_3]^2}{[\text{N}_2] [\text{H}_2]^3}$$



$$K = \frac{[\text{H}_2\text{O}]^2 \cdot [\text{CO}_2]}{[\text{CH}_4] \cdot [\text{O}_2]^2}$$

Chemical Reactions & Equilibrium

Now, let's do some math with these chemical equations:



What is the equilibrium value for the above equation if we measured the following at equilibrium:

Hydrogen gas exhibits a partial pressure of 2 atm.

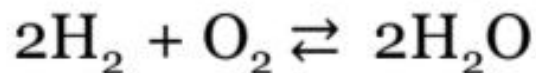
Oxygen gas exhibits a partial pressure of 3 atm.

Water vapor exhibits a partial pressure of 6 atm.

$$K = \frac{[\text{H}_2\text{O}]^2}{[\text{H}_2]^2 [\text{O}_2]} = \frac{6^2}{2^2 \cdot 3} = \frac{36}{4 \cdot 3} = \frac{36}{12} = 3$$

Chemical Reactions & Equilibrium

Using the same chemical equation,



$$\underline{\underline{K=3}}$$

12 atm
H₂O

1) What is the water vapor's pressure at equilibrium if we started with 4 atm of hydrogen gas and 3 atm of Oxygen gas?

$$K = \frac{[\text{H}_2\text{O}]^2}{[\text{H}_2]^2 [\text{O}_2]}$$

$$3 = \frac{x^2}{4^2 \cdot 3}$$

2) What is the hydrogen's pressure at equilibrium if we started with 3 atm of water vapor and 1/3 atm of oxygen gas?

$$3 = \frac{x^2}{4^2 \cdot 3} \Rightarrow x^2 = 3^2 \cdot 4^2 \Rightarrow x = 3 \cdot 4 = 12$$

Chemical Reactions & Equilibrium

$3 \cdot 4^2$

$$3 = \frac{x^2}{\cancel{3 \cdot 4^2}} \quad \times \cancel{3 \cdot 4^2}$$

$$\frac{3 \cdot 4^2 \cdot 3}{\cancel{3 \cdot 4^2}} = x^2$$

$$144 = x^2 \Rightarrow$$

$$\sqrt{x^2} = \sqrt{144}$$

$$x = 12$$