BERKELEY MATH CIRCLE

The Math of Chemistry

The Make-up of Atoms II: Electrons, Light & the EM Spectrum

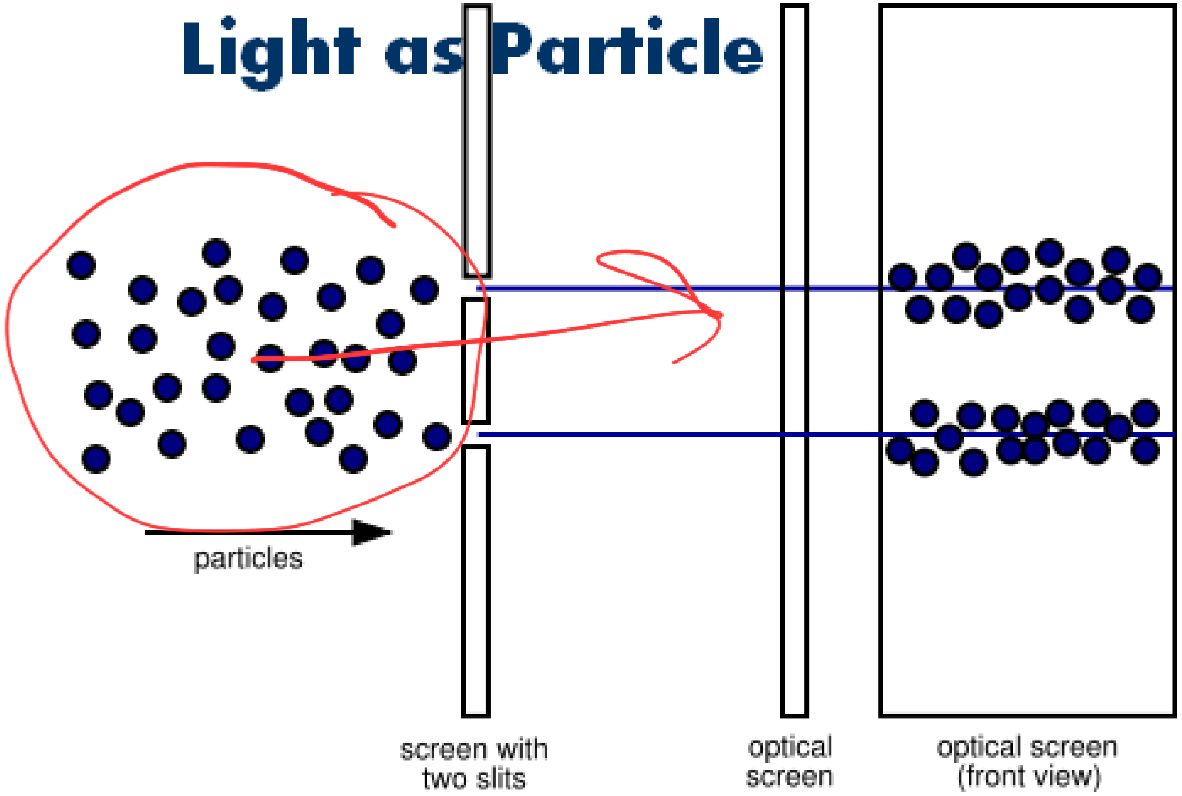
Instructor: Patricio Angulo

Let's transition to light and PHOTONS! We will relate this soon to Electrons ;)

Light is defined as both a WAVE and a PARTICLE! Dual Nature

As a PARTICLE, light exists in defined quantities known as Light Quanta or Photons, and these photons have energy associated with them (same is true for ANY moving object – think physics!). A photon is considered to be massless with no electric charge.

Energy



Double slit experiment (shows both qualities)

As a WAVE, light has properties of waves, including frequency (how often the cycle occurs) and wavelength (the length of each cycle). Mathematically, it looks like this:

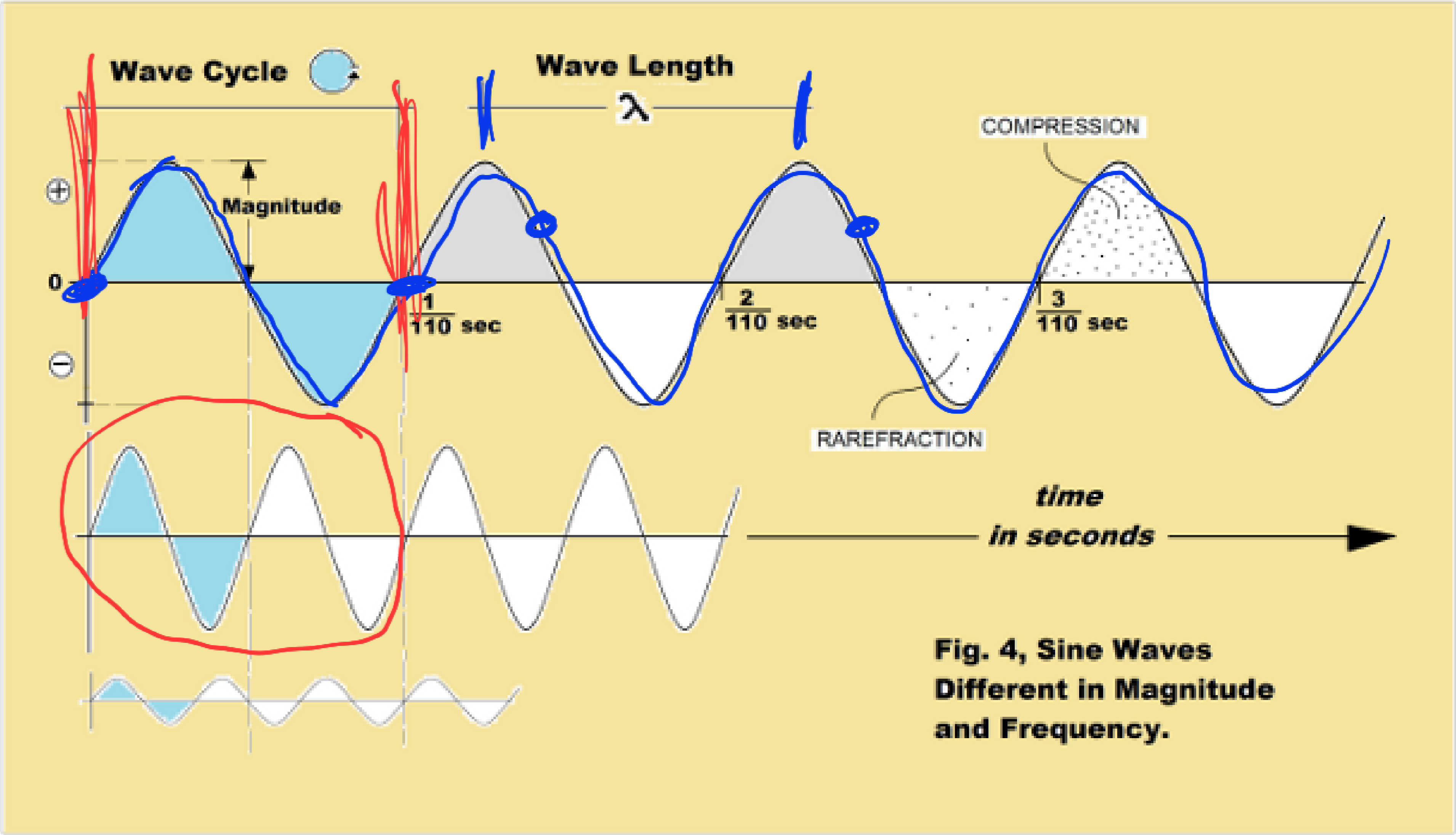


Fig. 4, Sine Waves Different in Magnitude and Frequency.

Anything that travels has energy, so therefore we can measure the energy of light, using this equation:

Planck's Equation → $E = hf$ → 3 variables

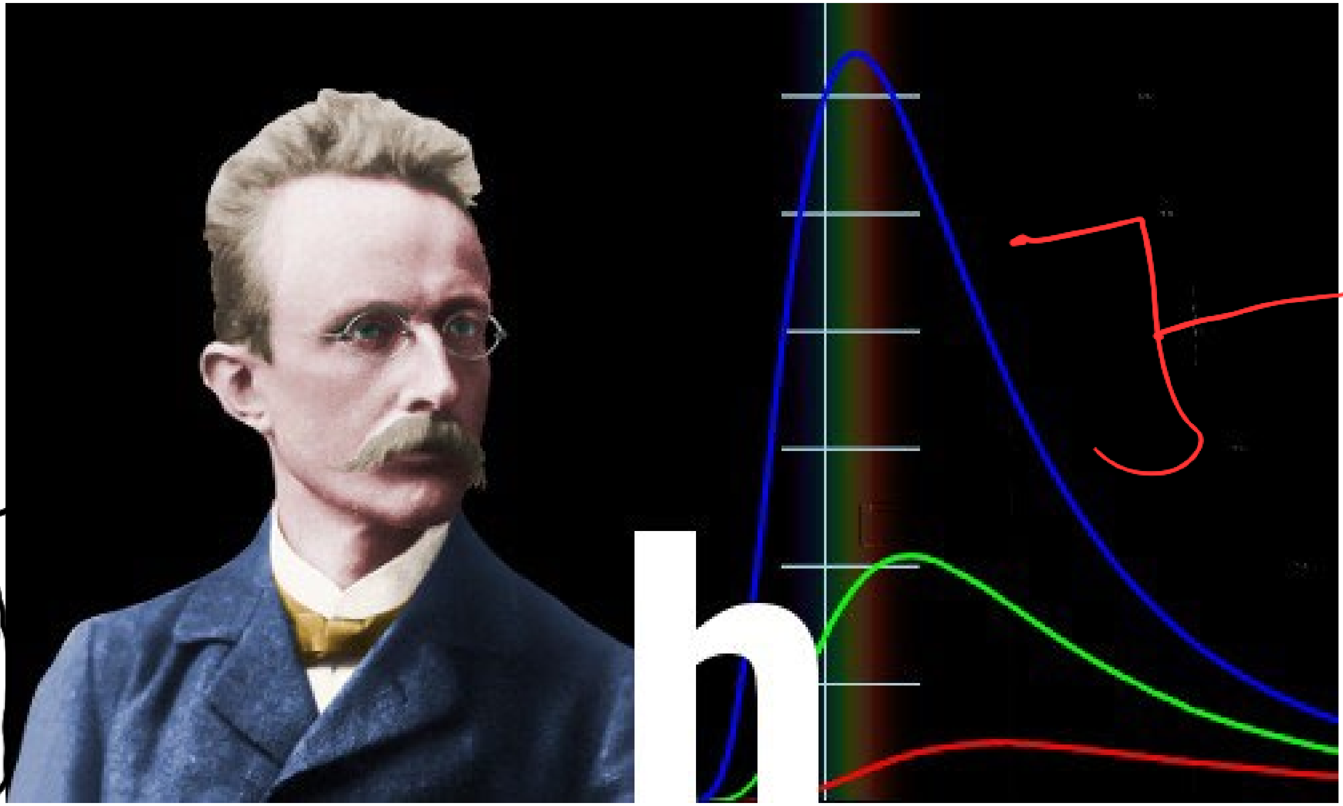
E = Energy of the Photon (Joules)

h = Planck's Constant = 6.626 x 10^{-34} J-s = **6.6 x 10^{-34} J-s** 6.6 x 10^{-34}

f = frequency of photon (hertz, cycles/second = 1/s) = how often the light "particles" keep coming

E vs f

frequency = $\frac{\text{cycles}}{\text{second}}$ = $\frac{1}{s}$ (hz)



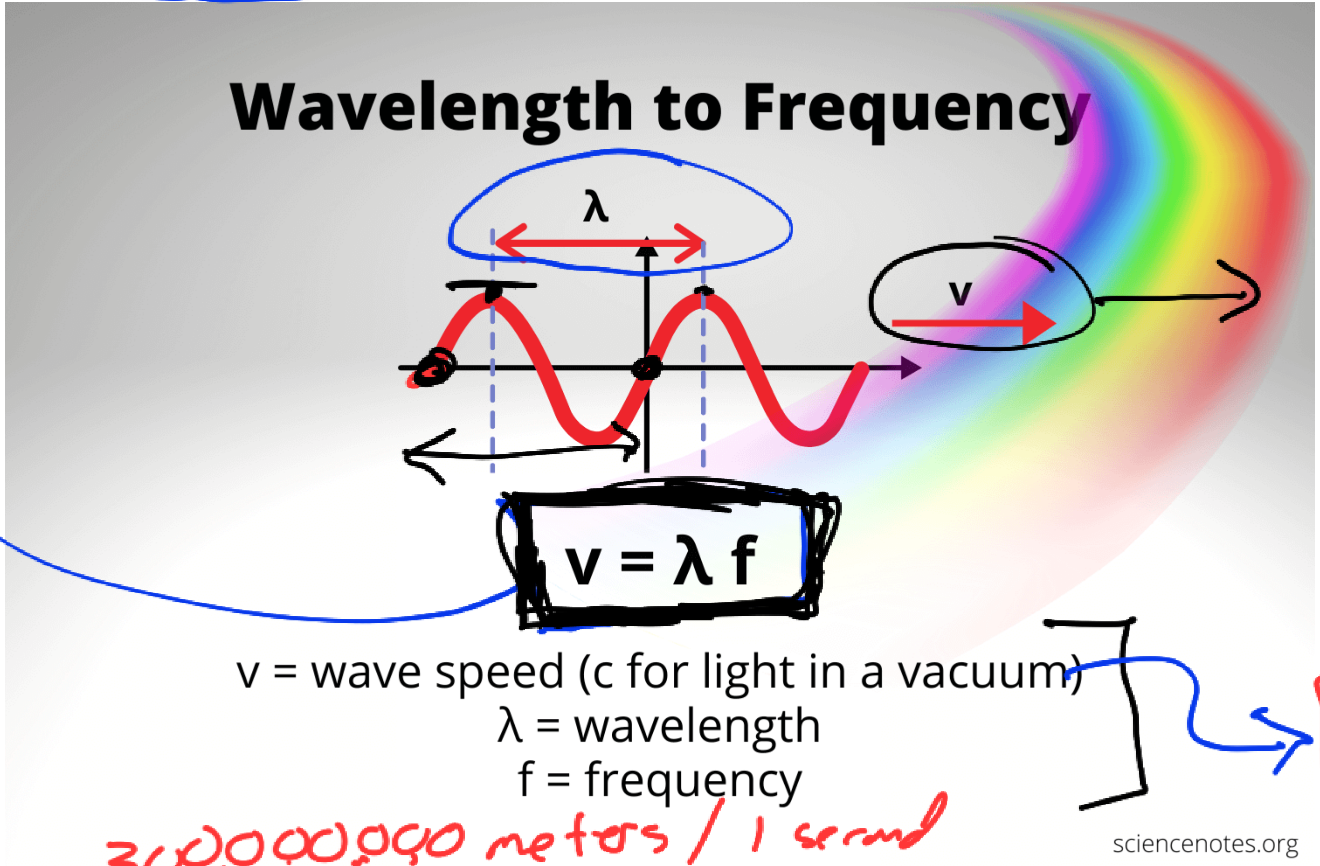
Different Energy Values for different forms of light

For waves, we think of a wave's frequency as related to it's wavelength, and this gives us the waves speed (this is true for any constant wave)!

Speed of wave = frequency of wave (f) x wavelength (λ)

Speed = v = meters/sec
frequency (f) = as the previous page, cycles/second (1/s) = how often the wave passes through (peak to peak)
wavelength = λ (lambda), measured in meters

$\frac{1}{s}$ = hertz (hz)



velocity ↓ speed ↓ How fast a wave travels

$v = \lambda \cdot f$

3×10^8 m/s

300,000,000 meters / 1 second

Going back to photons and light, we therefore have two items to consider:

Speed of Light

Speed = frequency (f) x wavelength (λ)

c = frequency (f) x wavelength (λ)

$c = f \times \lambda$

Where c = speed of light = 3×10^8 meters/second – Super fast!

$v = \lambda \cdot f$

$f = \frac{v}{\lambda}$

c

&

Plank's Equation

E = hf =

$E = h \cdot f$

Substituting through, we get:

$E = hF = E = h \cdot \frac{v}{\lambda} = \frac{hv}{\lambda}$

substitution

E = hf = hc/λ

Why does this matter?

1. Properties of different light types can be studied!
2. Fun with exponent math! We can review as needed

$$E = hf = hc/\lambda$$

$E = h \cdot f$ $E = \frac{h \cdot c}{\lambda}$

vel of wave = $\lambda \cdot f$

Energy and frequency are DIRECTLY related $E = h \cdot f$

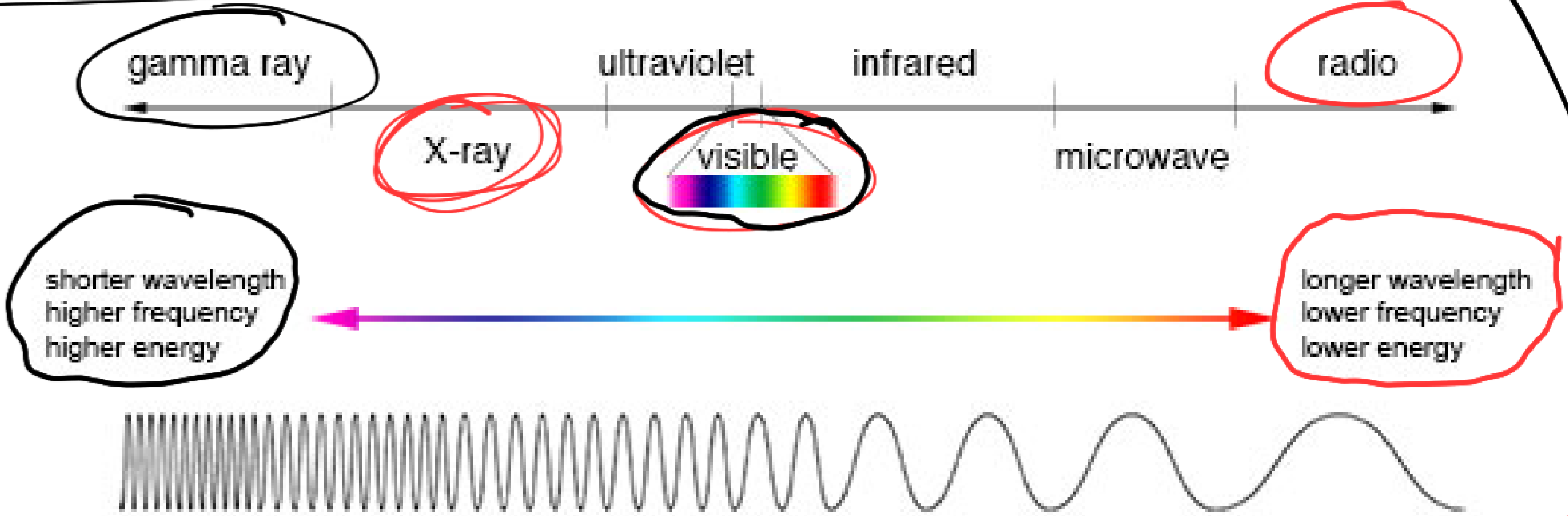
Energy and wavelength are INVERSELY related $E = \frac{hc}{\lambda}$

↑ Energy = ↑ frequency = ↓ wavelength

↓ Energy = ↓ frequency = ↑ wavelength

EM Spectrum = Electro-Magnetic Spectrum

↑E



↓E

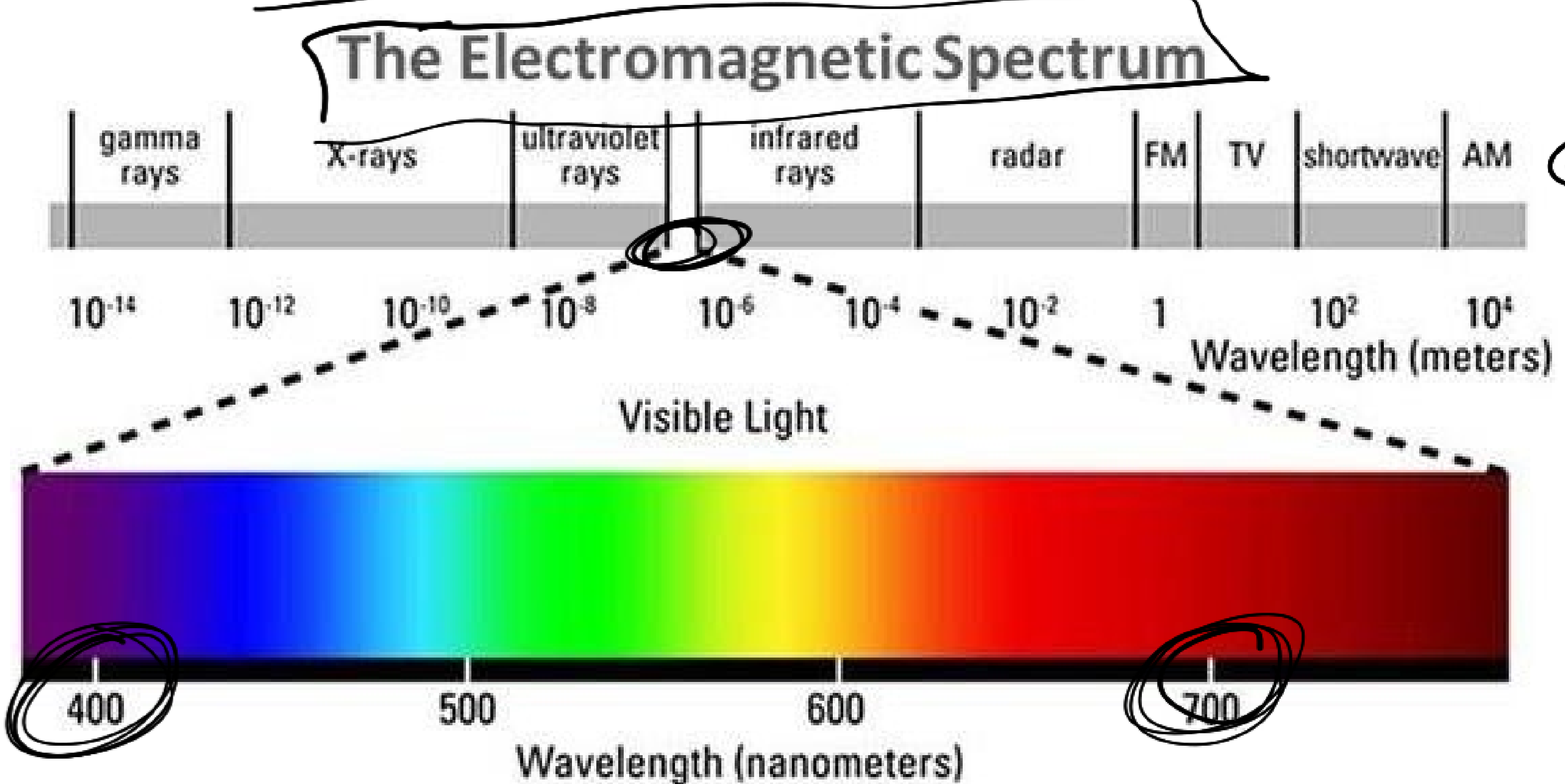
All light types have specified ranges for frequency and wavelength.
Commonly, wavelength is used to describe light.
The light we see, **visible light**, has wavelengths of 4×10^{-7} meters to 7×10^{-7} meters.

→ purple
↳ red

To measure visible light, we normally use nanometers:

1 meter = 1×10^{9} nanometers

Purple = 400 nm = 4×10^{-7}meters
Red = 700nm = 7×10^{-7}meters



Math Problems!

Pick your favorite rainbow color (ROYGBV) from this chart:

Color	Wavelength	Frequency	Energy
Violet	400nm		
Blue	450nm		
Green	500nm		
Yellow	550nm		
Orange	600nm		
Red	700nm		

Calculate your favorite visible light photon's frequency and Energy using $E = hf = hc/\lambda$.

For reference:

$h = 6.6 \times 10^{-34}$ J-s

$c = 3 \times 10^{8}$ m/s

$1 \text{ m} = 1 \times 10^{9}$ nm

This is a multi-step problem. Try as much mental math instead of using a calculator. Rounding off towards the final steps is acceptable ;)

Answer (1 of 6 possibilities)!

My favorite color is Green!

The 1st step is to convert from nanometers to meters:
Green = 500 nm

$500nm \times 1m/1 \times 10^9\ nm = 500 \times 10^0/10^9 = \mathbf{500 \times 10^{-9}} = 5 \times 10^{-7}\ m$
OR
$500nm \times 1m/1 \times 10^9\ nm = 5 \times 10^2\ nm \times 1m/\ 10^9\ nm = 5 \times 10^2\ /\ 10^9 = 5 \times 10^{-7}\ m$

The 2nd step is to use our fun, new equations!

To give us the energy, use **E = hc/λ**

$\mathbf{E = hc/\lambda} = (6.6 \times 10^{-34}\ J\text{-}s)(3 \times 10^8\ m/s)/\ 5 \times 10^{-7}\ m\ )$

$(6.6)(3)/(5) \times (10^{-34})(10^8)/(10^{-7})$

$19.8/5 \times (10^{-26})/(10^{-7}) \approx 4 \times 10^{-19}$

3.96×10^{-19} Joules $\approx 4 \times 10^{-19}$ Joules

Answer (1 of 6 possibilities) Continued . . . .

Then, use your answer with this equation: E = hf

E = hf

3.96×10^{-19} Joules = (6.6×10^{-34} J-s) x f

$f = 3.96 \times 10^{-19} / 6.6 \times 10^{-34}$

$f = 6 \times 10^{14}$

E (previous page)

h = constant

f = solve for this

Whew!

Color	Wavelength	Frequency (1/s)	Energy (J)
Violet	400nm	7.5×10^{14}	4.95×10^{-19}
Blue	450nm	6.67×10^{14}	4.4×10^{-19}
Green	500nm	6×10^{14}	3.96×10^{-19}
Yellow	550nm	5.45×10^{14}	3.6×10^{-19}
Orange	600nm	5×10^{14}	3.3×10^{-19}
Red	700nm	4.29×10^{14}	2.83×10^{-19}