$\alpha\chi \geq |V|$

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Berkeley Math Circle
March 22, 2023

Question 1. How many cells can one pick in a 100×100 toroidal grid with no two touching each other?

Question 2. In Balifornia, license plates must be of the form 1ABC234, and to avoid errors, no two plates are allowed to be within a character of each other. What is the maximum number of Balifornia license plates?

Question 3. We say that two binary strings of length n are *similar* if it's possible to delete one bit from each to obtain the same length $n-1$ string. (For instance, if you delete the 2nd bit from 110010, you get the string 10010.)

Show that there exist at least $2^n/(n+1)$ binary strings of length n such that no two strings are similar.

Question 4. Show that among the binary strings of length $2^n - 1$ there are:

a. 2^{2^n-1-n}, any two of which differ in more than two bits,

b. 2^n, any two of which differ in the majority of their bits.

Question 5. A country with n cities has some two-way roads connecting certain pairs of cities. Someone notices that if the country is split into two parts in any way, then there would be at most kn roads between the two parts (where k is a fixed positive integer). What is the largest integer m (in terms of n and k) such that there is guaranteed to be a set of m cities, no two of which are directly connected by a road?

Question 6 (2021 IMO Shortlist C8)**.** Determine the largest integer N for which there exists a table T of integers with N rows and 100 columns that has the following properties:

(i) Every row contains the numbers 1, 2, ..., 100 in some order.

(ii) For any two distinct rows r and s, there is a column c such that $|T(r,c) - T(s,c)| \geq 2$. (Here $T(r,c)$ is the entry in row r and column c.)