Polyomino Tilings

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Definition 1. A *polyomino* is a finite connected plane figure formed by joining unit squares edge to edge.

Classic

Problem 1. Can a chessboard with opposite corners removed be tiled with dominoes?

Problem 2. Can a chessboard with one corner removed be tiled with L-trominoes?

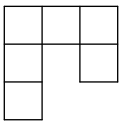
Problem 3 (MC2 P6)**.** Can a chessboard with all corners removed be tiled with L-tetrominoes?

Difficult

Problem 4 (2014 C4)**.** Assume that P can be tiled with S-tetrominoes. Prove that any tiling of P using only S- and Z-tetrominoes always uses an even number of Z-tetrominoes.

Problem 5. Assume that P can be tiled with only O-tetrominoes, and also that it can be tiled with only S-tetrominoes and Z-tetrominoes. Show that it is not simply connected[1].

Problem 6 (2004 IMO 3)**.** Determine all rectangles that can be tiled with the "hook" shape shown below.



Bonus

Problem 7. For which n can all polyominoes with area n tile a rectangle?

Problem 8. Show that if a set of tiles can tile the first quadrant, they can tile the whole plane.

Problem 9 (de Bruijn)**.** If a box can be filled with $1 \times 2 \times 4$ bricks, must there be a way to fill the box with bricks so that they all have the same orientation?

[1] A shape is not simply connected if it has a hole.