BMC Fall 2022
Beginners I Group

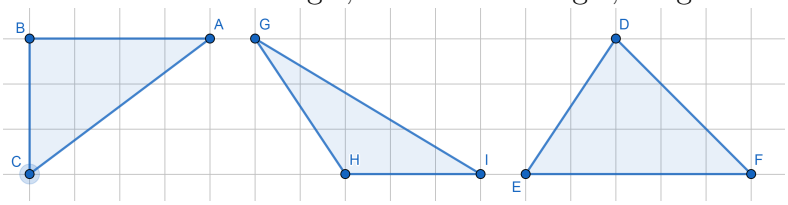
Polygon Decompositions - Part I

By Harry Main-Luu

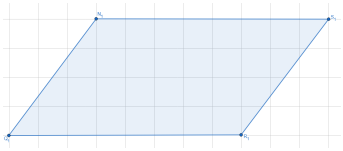
I. Useful Knowledge

The concepts and formulas below can become useful when we need to prove that our puzzles work the way they do. If you don't know any of the following formula, make sure you raise your hand and ask! If you do know the formula, then ask yourself if you can prove that the formula is correct, that is "why is the formula true?"

1. Calculating areas of familiar shapes:
   a) Area of a rectangle, a square.
   b) The difference between a rectangle and a square.
   c) Area of an acute triangle, an obtuse triangle, a right triangle?



   d) Area of a parallelogram.



   e) The difference between a parallelogram and a rectangle.

2. Standard Euclid construction with compass and straight-edge:
   If you are not quite yet familiar with how to perform the following constructions using a compass and a straight-edge, you can take for granted that these can be done.
   a) Find the midpoint of a segment.
   b) Draw a line through a given point perpendicular to another line.
   c) Constructing a right triangle with a given hypotenuse.

3. The sum of three interior angles of a triangle.

4. Basic properties of an isosceles triangle.

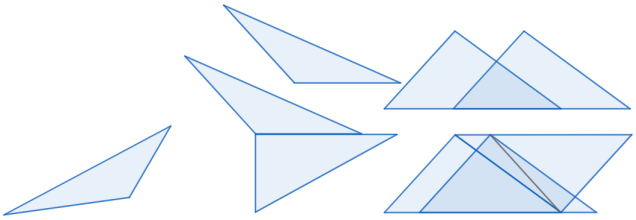
5. Harry's childhood favorite theorem: Pythagorean Theorem.

6. Some familiarity with square roots are preferred, but not required.

II. Warm-ups and Preambles

1. What does it mean that two planar shapes are congruent?

2. Find as many congruent triangles as you can in the picture below.



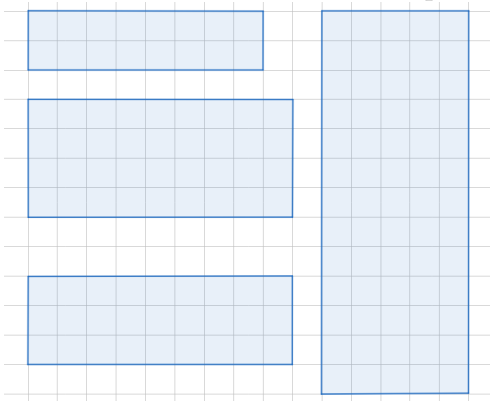
III. The Squaring Problems

For the first two sessions, we will focus on one type of problem:

> Can we cut a certain shape into finitely many pieces and then glue it back into a perfect, beautiful square?

You may want to ask: Why square? Why not any other shape?
For now, one answer is: because squares are one of the nicest shapes, and we like nice things. :)

1. Let's start with the closest shape to a square, which in this case is a rectangle.



   a) You should notice by now that some rectangles are harder than others. Maybe we should ask which ones we can cut and glue back into a square and which ones we can not?
   The most satisfying yet annoying answer is:

   b) Then how are we going to cut a random rectangle?