BMC Spring 2022 Int II

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1 Rates of Change

AROC and IROC

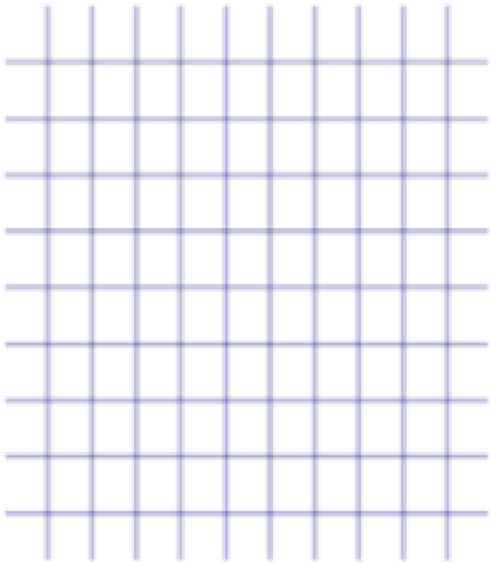
1. A train travels from city A to city B. It leaves city A at 10:30 am and arrives at city B at 1:30 pm. The distance between the cities is 150 miles. What was the average velocity of the train in miles per hour?
2. Find the average rate of change of the function $R(t) = \sqrt{2t-7}$ as t changes from 1 to 9.
3. Let $f(x) = 2x^2 - 3x$ Write an expression for the average rate of change of $f(x)$ from $x = 3$ to $x = 3 + h$.
4. Let $g(t) = cos(t)$. Write an expression for the average rate of change as t changes from 4 to $4 + h$.
5. Find the instantaneous rate of change of the function $f(t) = t^2 + 3$ at $t = 2$ by calculating the limit of the average rate of change. Write the rate expression you used.
6. A marble is dropped from the top of a 100-m-high tower. Its height above ground after t seconds is given by the function $h(t) = 100 - 4.9t^2$ meters. Write a rate expression and calculate how fast the marble is falling 2 seconds after it is dropped?.
7. Expand and simplify $\frac{f(7+h)-f(7)}{h}$ for $f(x) = x^2$
8. Consider a triangle with base = x and height = 2x. Write an expression for the Area of the triangle as a function of x. Write a rate expression for the area. Find the instantaneous rate of change of the area of the triangle when x = 5.

9. Given the function $f(x) = \sqrt{x}$

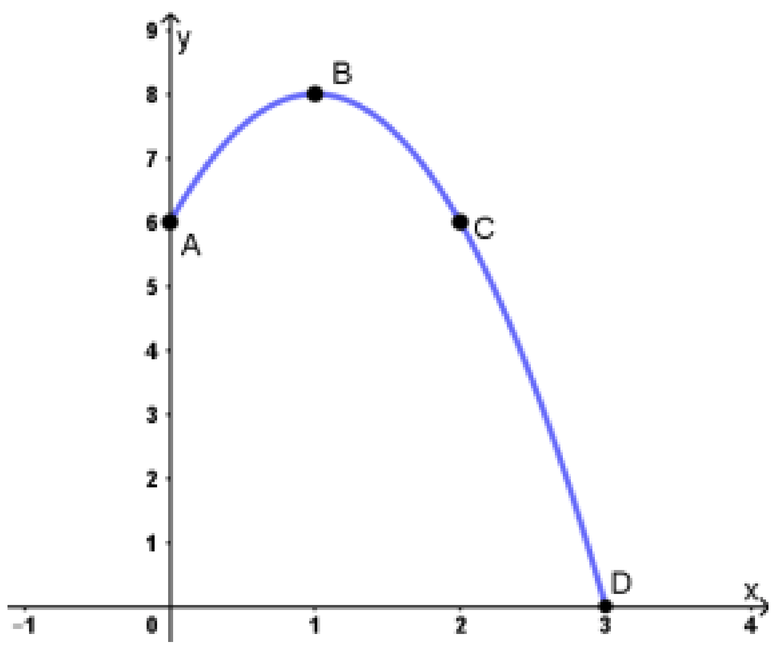
   (a) Write a rate expression and find the slope between $(1, f(1))$ and $(9, f(9))$

   (b) Write a rate expression and find the slope of the tangent at $(4, f(4))$

   (c) Write the equation of the tangent line. Graph $f(x)$ and the tangent line.



10. Given $f(x) = -2x^2 + 4x + 6$, Draw slope triangles, write slope expressions and calculate the AROC between these points



   (a) From A to B

   (b) From B to C

   (c) From C to D

   (d) Between which two of these points will the slope = 0?

   (e) The lines between any two points on the curve is called a secant line. A line touching a point on the graph without cutting through the curve is called a tangent line. At which point on this curve will the tangent line have a slope = 0? Draw this tangent line.

11. Write slope expressions and calculate the average rate of change of $f(x) = e^x$ over the following intervals

$$[-1, \ -0.999] \qquad\qquad [0, 0.001] \qquad\qquad [1, \ 1.0001]$$

(a) Calculate the values

$$e^{-1} = \qquad\qquad e^0 = \qquad\qquad e^1 = \qquad\qquad e^2 =$$

(b) Write a conjecture about the relationship between the rate of change of $f(x) = e^x$ at a given point and the value at that point.

12. Expand and simplify each of the following slope expressions

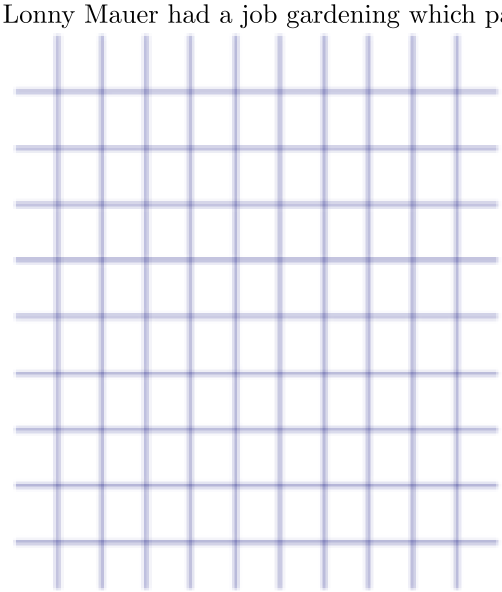
a) $\dfrac{(x+h)^2 - x^2}{(x+h) - x}$ b) $\dfrac{5(x+h)^2 - 5x^2}{(x+h) - x}$ c) $\dfrac{(x+h)^3 - x^3}{(x+h) - x}$

13. For each of the expressions find the limit as $h \to 0$

a) $\lim\limits_{h \to 0} \dfrac{(x+h)^2 - x^2}{(x+h) - x} =$ b) $\lim\limits_{h \to 0} \dfrac{5(x+h)^2 - 5x^2}{(x+h) - x} =$ c) $\lim\limits_{h \to 0} \dfrac{(x+h)^3 - x^3}{(x+h) - x} =$

2 The Area Problem

14. Lonny Mauer had a job gardening which paid \$6.00 per hour



(a) Label the scales to cover a 16-hour workday and up to a \$15 per hour pay rate.

(b) Lonny works for 8 hours on Saturday. Graph a horizontal line showing his pay rate for the 8 hours

(c) Color the rectangle below the horizontal line in (b) and calculate the area of the rectangle.

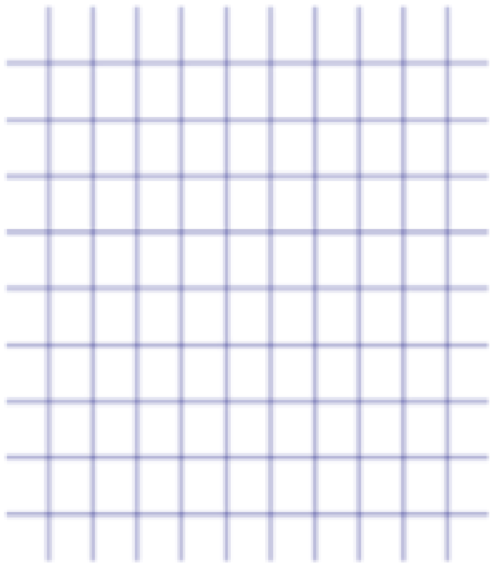
(d) What is the meaning of the area in (c)?

(e) . Add a rectangle from 8 to 12 hours, showing Lonny earning time-and-a-half. Calculate the area of this rectangle.

(f) Add a rectangle from 12 to 16 hours, showing Lonny earning double-time-and-a-half. Calculate the area of this rectangle.

(g) How much did Lonny earn for the grueling 16-hour workday?

15. Mario Andretti traveled at 60 miles per hour for two hours, then sped up at a constant rate from 60 mph to 100 mph in a one hour time period.



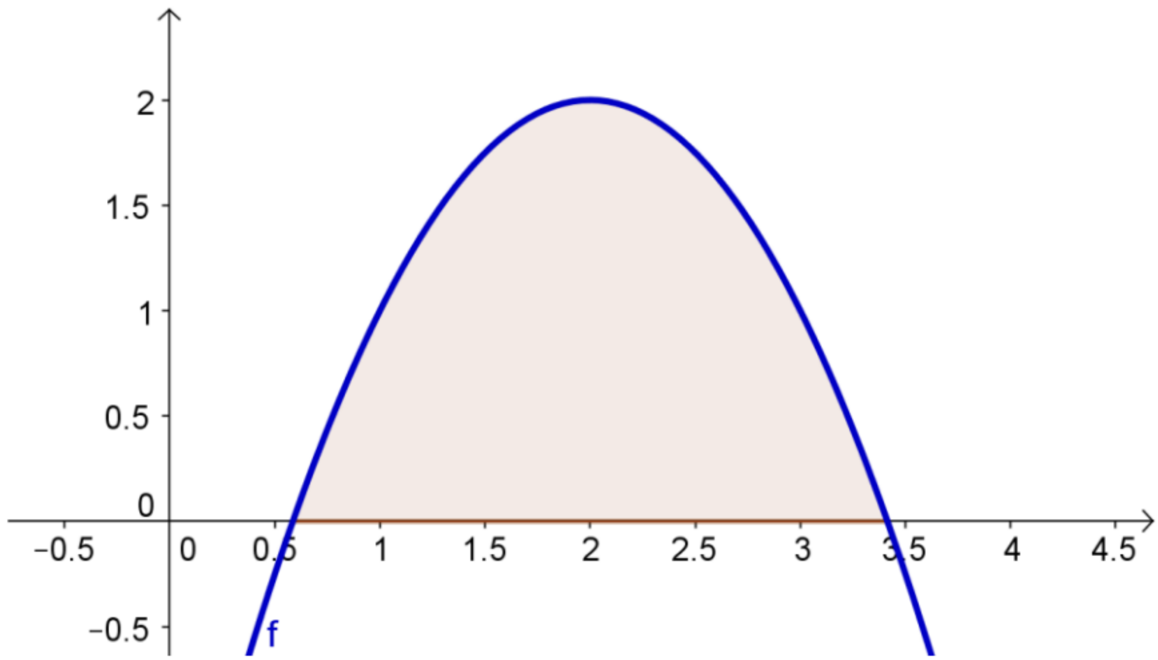
(a) Sketch the graph of Mario's speed vs time

(b) Divide the region between the graph and the x-axis into two geometric regions. Calculate the area of each region.

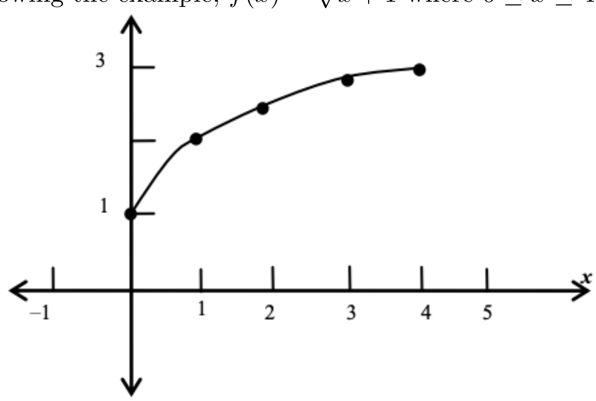
(c) Total Area =

(d) What is the meaning of the total area in (c)?

16. Estimating the Area Under a Curve Describe several methods for estimating the area of the shaded region. Show your calculations. Which method was easiest? Which method do you think was most accurate? Why?



17. Following the example, $f(x) = \sqrt{x} + 1$ where $0 \leq x \leq 4$ sketch 4 right-endpoint rectangles



Rectangle #	Base Δx	Right Endpt, x_i	Height, $f(x_i)$	$A = f(x_i) \cdot \Delta x$
R_1				
R_2				
R_3				
R_4				
			Total Area (Right) Estimate	

18. Conclusion from above: How does the actual area relate to the Right Estimate? How might you use the Right and Left estimates to get a better approximation of the exact area?

19. Determine the following limits for the graph below

a) $\lim_{x \to -4} f(x) =$ ___

b) $\lim_{x \to -1^-} f(x) =$ ___

c) $\lim_{x \to -1^+} f(x) =$ ___

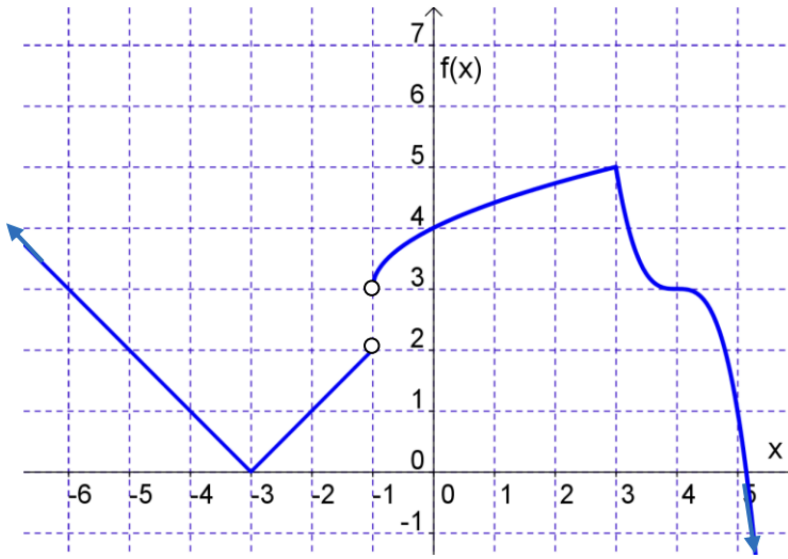
d) $\lim_{x \to 3^-} f(x) =$ ___

e) $\lim_{x \to 3^+} f(x) =$ ___

f) $\lim_{x \to 4} f(x) =$ ___

g) $\lim_{x \to -\infty} f(x) =$ ___

h) $\lim_{x \to \infty} f(x) =$ ___



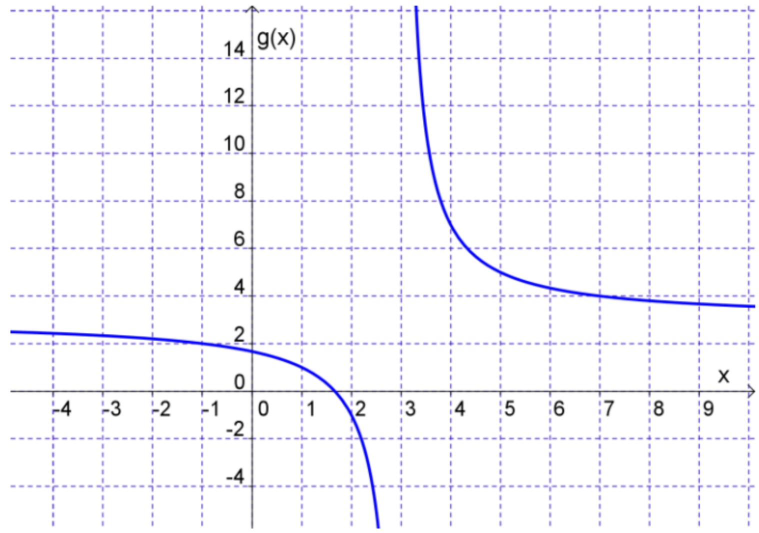
20. Determine the following limits for the graph below

$$g(x)=\frac{x^2-1}{(x-1)(x-3)}+2$$

The graph of *g(x)* is shown. Add arrows to the ends to show the direction the graph goes.

Don't depend on just the graph to determine the following limits.

Check by calculating points on the graph.



a) $\lim_{x\to 0} g(x) =$ ___

b) $\lim_{x\to 3^-} g(x) =$ ___

c) $\lim_{x\to 3^+} g(x) =$ ___

d) $\lim_{x\to 1} g(x) =$ ___

e) $\lim_{x\to 5} g(x) =$ ___

f) $\lim_{x\to -\infty} g(x) =$ ___

g) $\lim_{x\to \infty} g(x) =$ ___

On the graph of *g(x)* above, determine and draw the following

h) the hole in the graph is at ___________

i) the vertical asymptote is at __________

j) the horizontal asymptote is at ________