BMC Intermediate II March 9, 2022 Pratima Karpe

1) A square pyramid and a tetrahedron with all the side lengths 1 are “fused”. How many faces?

2) A right prism with height h has bases that are regular hexagons with sides of length 12. A vertex A of the prism and its three adjacent vertices are the vertices of a triangular pyramid. The dihedral angle (the angle between the two planes) formed by the face of the pyramid that lies in a base of the prism and the face of the pyramid that does not contain A measures 60 degrees. Find h^2.

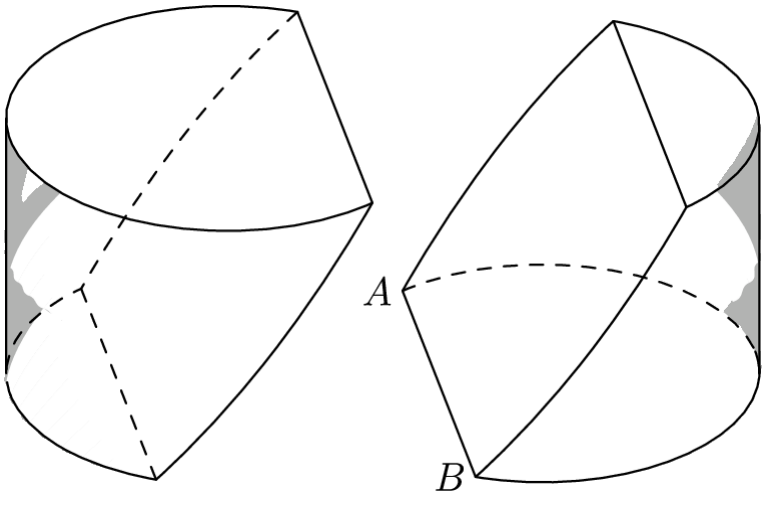
3) Kangaroo Math

30. A cube of side length 3 consists of 15 black and 12 white unit cubes. In the diagram five of the six faces of the big cube can be seen.

Which of the regions shown below is the 6th face of the big cube?

(A)  (B)  (C)  (D)  (E) 

4) A block of wood has the shape of a right circular cylinder with radius 6 and height 8, and its entire surface has been painted blue. Points A and B are chosen on the edge of one of the circular faces of the cylinder so that $\overset{\frown}{\mathrm{AB}}$ on that face measures 120°. The block is then sliced in half along the plane that passes through point A, point B, and the center of the cylinder, revealing a flat, unpainted face on each half. The area of one of these unpainted faces is $a \cdot \pi + b\sqrt{c}$, where a, b, and c are integers and c is not divisible by the square of any prime. Find $a + b + c$.

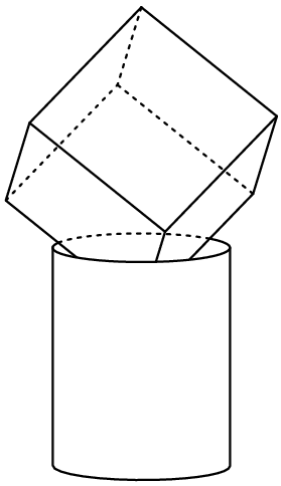


Credit to Royalreter1 and chezbgone2 for the diagram

5) Segments $\overline{AB}, \overline{AC},$ and $\overline{AD}$ are edges of a cube and $\overline{AG}$ is a diagonal through the center of the cube. Point P satisfies $PB = 60\sqrt{10}, PC = 60\sqrt{5}, PD = 120\sqrt{2},$ and $PG = 36\sqrt{7}$. What is PA?

6) Tetrahedron $ABCD$ has $AD = BC = 28$, $AC = BD = 44$, and $AB = CD = 52$. For any point X in space, suppose $f(X) = AX + BX + CX + DX$. The least possible value of $f(X)$ can be expressed as $m\sqrt{n}$, where m and n are positive integers, and n is not divisible by the square of any prime. Find $m + n$.

7) A cylindrical barrel with radius 4 feet and height 10 feet is full of water. A solid cube with side length 8 feet is set into the barrel so that the diagonal of the cube is vertical. The volume of water thus displaced is v cubic feet. Find v^2.



8) A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into the two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers, a and c are relatively prime, and b is not divisible by the square of any prime. What is $a + b + c$?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 14$

Problems from AIME, Kangaroo, SAT, AMC