Homework 1

1) Find integers a, b, c such that $a|bc$ but $a \nmid b$ and $a \nmid c$.

2) Find the unique integers q and r guaranteed by the division algorithm for each pair of integers below.

a) $a = 47$, $b = 6$

b) $a = 281$, $b = 13$

c) $a = 343$, $b = 49$

3) Prove or disprove the following statements:

a) If a, b, c, and d are integers such that $a|b$ and $c|d$ then $a + c|b + d$.

b) If a, b, c, and d are integers such that $a|b$ and $c|d$, then $ac|bd$.

4) Let $n \in \mathbb{Z}$ be positive. Prove that $n|(n+1)^n - 1$.

5) a) Let n be an integer. Prove that $3|n^3 - n$.

b) Let n be an integer. Is it true that $4|n^4 - n$? Provide a proof or find a counterexample.

6) a) Given the definition of even and odd integers from class (an even is an integer divisible by 2, an odd is an integer not divisible by 2), show that every integer is of the form $2m$ for some integer m, and every odd integer is of the form $2m + 1$ for some integer m.

b) Prove that the sum and product of two even integers are even.

c) Prove that the sum of two odd integers is even, and the product of two odd integers is odd.

7) Prove that 2 is the only even prime number.

8) Prove or disprove the following statement:

There are infinitely many primes p for which both $p + 2$ and $p + 4$ are also prime numbers.

9) Prove that every integer greater than 11 can be expressed as a sum of two composite (i.e. not prime and not 1) numbers.

10) How difficult was this homework? How long did it take?