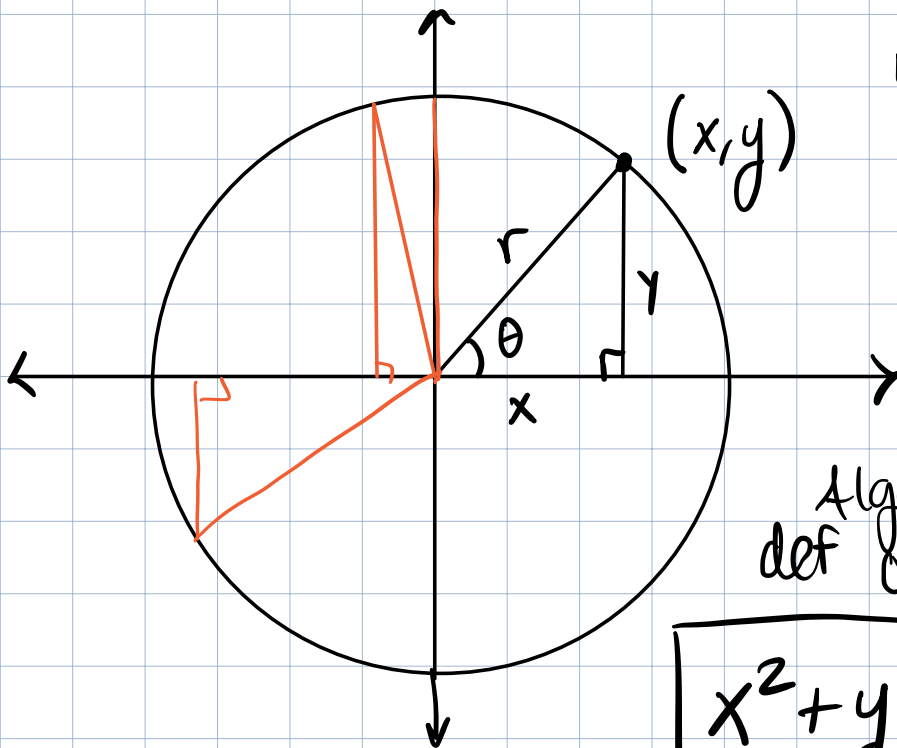


circle is not
a function



we can break
down a circle
using right
triangles.

Algebraic
def of a circle

$$x^2 + y^2 = r^2$$

Def

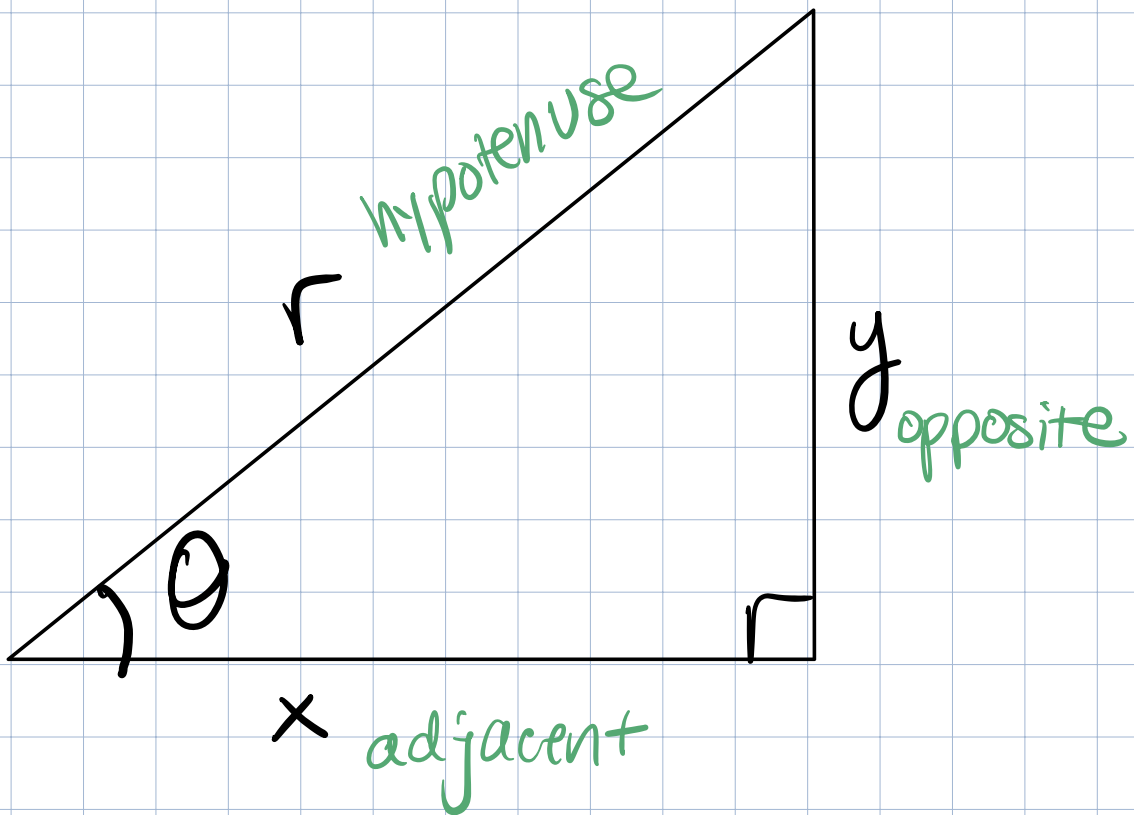
Trigonometry
of triangles

is the study of

we have 6 trigonometric functions that are defined as a function of θ , and the ratio of two sides.

- ① sine
- ② cosine
- ③ tangent

- ④ cosecant
- ⑤ secant
- ⑥ cotangent



$$\textcircled{1} \sin(\theta) = \frac{y}{r}$$

$$\textcircled{2} \cos(\theta) = \frac{x}{r}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\textcircled{3} \tan(\theta) = \frac{y}{x}$$

$$\textcircled{4} \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\csc(\theta) = \frac{r}{y}$$

$$\textcircled{5} \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\sec(\theta) = \frac{r}{x}$$

$$\textcircled{6} \cot(\theta) = \frac{1}{\tan(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\cot(\theta) = \frac{x}{y}$$

Generally

S O C A T O
H H A A

sine SOH

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

cosine CAH

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

tangent TOA

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

prove that

$$\text{if } \sin(\theta) = \frac{y}{r} \text{ and } \cos(\theta) = \frac{x}{r},$$

$$\text{then } \sin^2 \theta + \cos^2 \theta = 1.$$

Proof

$$\text{Given that } \sin(\theta) = \frac{y}{r},$$

$$\Rightarrow r \cdot \sin(\theta) = y$$

$$\text{Given that } \cos(\theta) = \frac{x}{r}$$

$$\Rightarrow r \cdot \cos(\theta) = x$$

$$x^2 + y^2 = r^2,$$

$$\cos^2(\theta) = \cos(\theta) \cdot \cos(\theta)$$

$$x = r \cos(\theta) \quad y = r \sin(\theta)$$

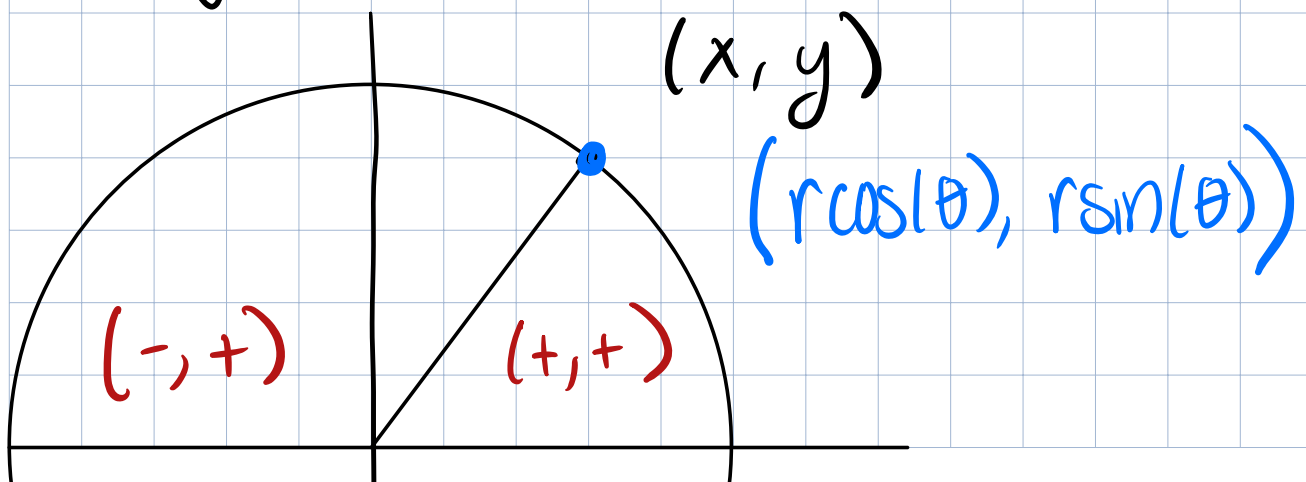
$$(r \cdot \cos(\theta))^2 + (r \cdot \sin(\theta))^2 = r^2$$

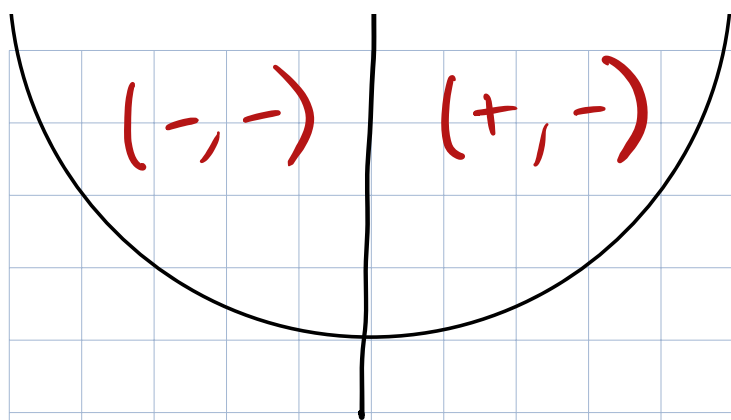
$$\frac{r^2 \cdot \cos^2(\theta) + r^2 \cdot \sin^2(\theta)}{r^2} = \frac{r^2}{r^2}$$

$$\cos^2(\theta) + \sin^2(\theta) = 1 \quad \square$$

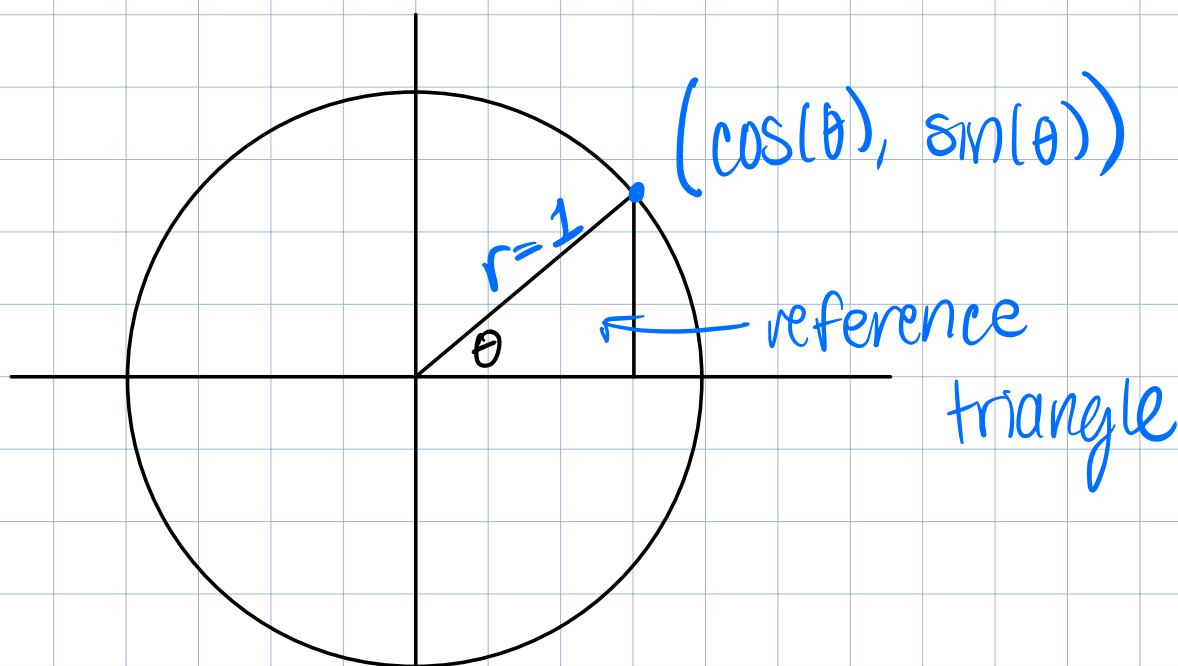
$$y = r \cdot \sin(\theta)$$

$$x = r \cdot \cos(\theta)$$

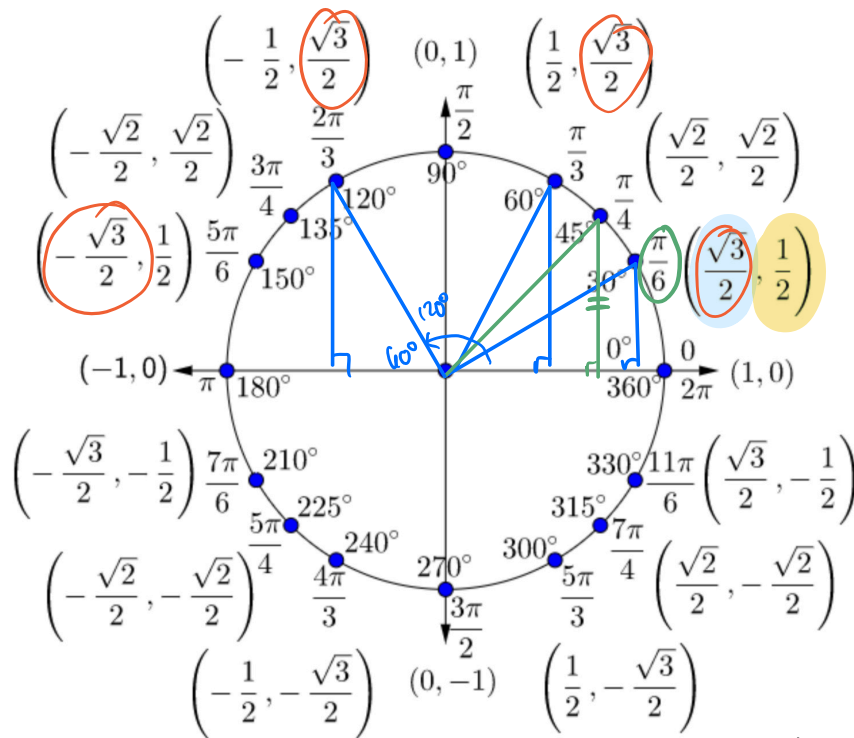




A circle w/a radius of 1,
↳ unit circle.

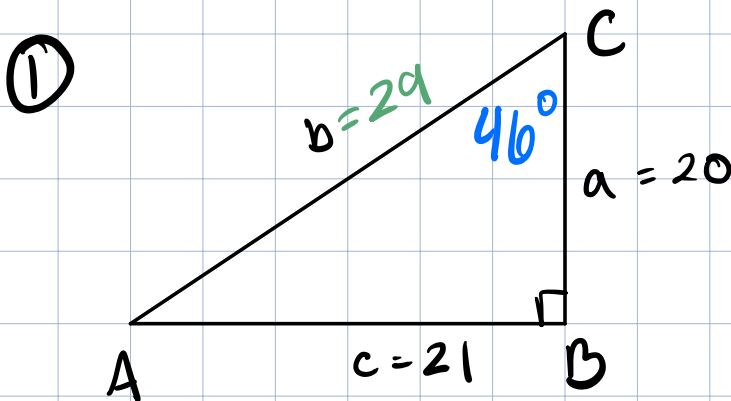


$$(\cos(\theta), \sin(\theta))$$



$$\sin(90^\circ - \theta) = \cos(\theta)$$

- what do you notice?
- what patterns do you see?
- similarities and differences between quadrants?



side b

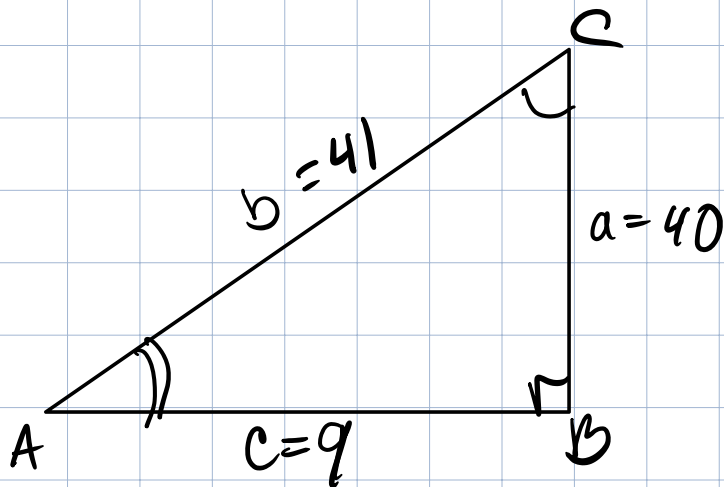
pythagorean thm.

$$20^2 + 21^2 = b^2$$
$$b = 29$$

$$\cos^{-1}(\cos(C)) = \left(\frac{20}{29}\right)^{\cos^{-1}} \Rightarrow C^\circ = \cos^{-1}\left(\frac{20}{29}\right)$$
$$C^\circ \approx 46^\circ$$

$$A^\circ = 44^\circ$$

③



$$b = 41$$

$$\sin(A) = \frac{40}{41}$$

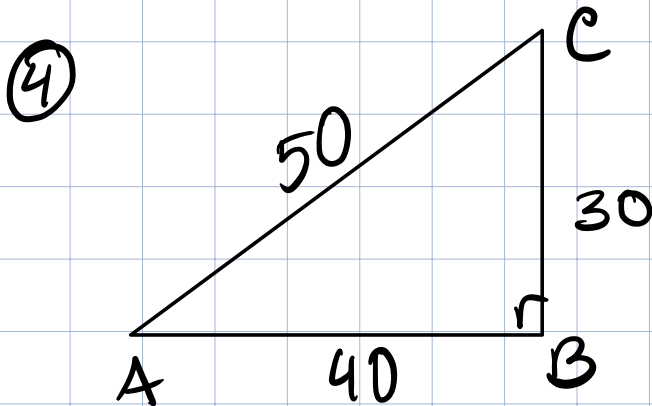
$$\cos(A) = \frac{9}{41}$$

$$\cos(C) = \frac{40}{41}$$

$$\sin(C) = \frac{9}{41}$$

$$\underline{\underline{30^\circ}} - \underline{\underline{60^\circ}} - 90^\circ$$

$$\sin(90 - \theta) = \cos(\theta)$$



$$b = 50$$

$$\sin(A) = \frac{3}{5}$$

$$\cos(A) = \frac{4}{5}$$

$$\tan(A) = \frac{3}{4}$$

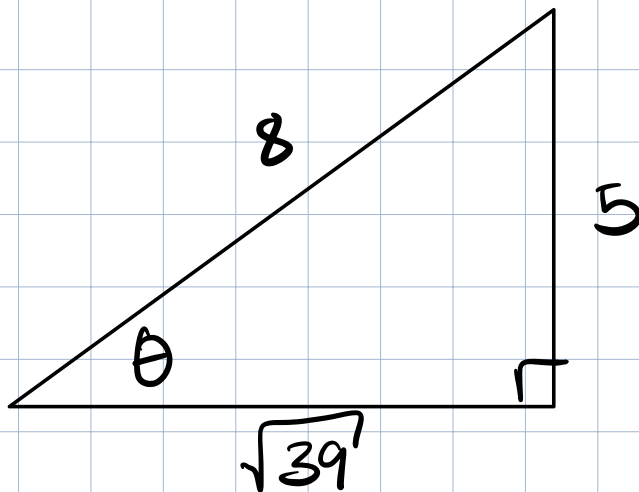
$$\sin(C) = \frac{4}{5}$$

$$C = \sin^{-1}\left(\frac{4}{5}\right)$$

$$C = 53^\circ$$

$$A = 37^\circ$$

⑤



$$\sin(\theta) = \frac{5}{8}$$

$$\cos(\theta) = \frac{\sqrt{39}}{8}$$

$$\tan(\theta) = \frac{5}{\sqrt{39}} = \frac{5\sqrt{39}}{39}$$

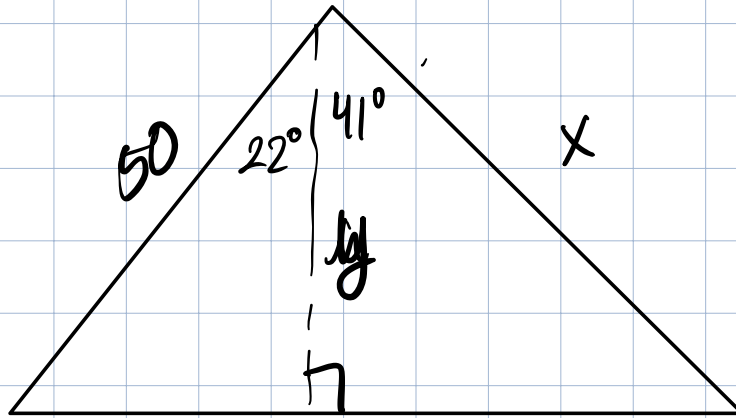
$$\csc(\theta) = 8/5$$

$$\sec(\theta) = 8/\sqrt{39}$$

$$\cot(\theta) = \frac{\sqrt{39}}{5}$$

$$= \frac{8\sqrt{39}}{39}$$

(6)



$$\cos(22^\circ) = \frac{y}{50}$$

$$50 \cdot \cos(22^\circ) = y$$