King Chickens[1] Handout #1 Elysée Wilson-Egolf

1. For each chicken in a flock, count the number of other chickens which that particular chicken pecks. Let K be the chicken with the highest peck count. (If there is a tie, let K be any one of the winners). Prove that K is a king. This shows that every flock has at least one king.

2. How can we arrange for a flock (of any given size) to have exactly one king?

3. If a chicken has the barnyard to itself, of course it is king. How many kings will there be in a flock with two chickens?

4. There are essentially two different possible pecking orders for a flock with three chickens. How many of the three chickens are kings in each case?

5. Find a way for a flock of four chickens to have exactly one or three kings. Then show that it is impossible for such a flock to have exactly two or four kings.

6. Suppose we have a flock of n chickens with exactly k kings. Show that in this case there exists a flock of n + 1 chickens which also has exactly k kings.

7. Construct a pecking order for a flock with an odd number of birds in which every chicken is a king.

8. Suppose we have a flock of n chickens in which every chicken is a king. Explain how to construct a flock of n + 2 chickens with the same property, so that every chicken is again a king.

9. Establish the following lemma: given a particular chicken C, if C is pecked by other chickens, then one of the chickens that pecks C must be a king.

10. Now prove that no flock can have exactly two kings.

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1 These materials taken from Sam Vandervelde's *Math Circle in a Box*, Chapter 7.