Invariants

Problem 1:

$0\ \cancel{0}\ \cancel{0}\ \cancel{0}\ \cancel{1}\ 1\ 1\ 1\ 0$

start: $0\ \cancel{0}\ \cancel{0}\ \cancel{0}\ \cancel{1}\ 1\ 1\ 0\ 1$

end:

$0\ 0\ 1\ 1$

$0\ 0\ 0\ 1\ 1$

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start: 0000111

end: 1

$$0+0+0+0+1+1+1=3$$

$$\cancel{0}+\cancel{0}+0+0+1+1+1+0=3$$

$$\cancel{0}+\cancel{0}+0+\cancel{0}+\cancel{1}+1+1+0+1=3$$

1 - 2

$\cancel{0}+\cancel{0}+0+\cancel{0}+\cancel{1}+\cancel{1}+\cancel{1}+0+1+0=1$

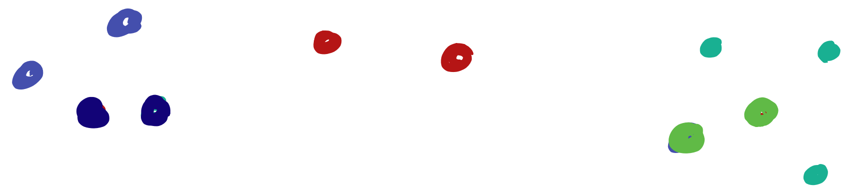
If we start with an odd sum,
we end up with the sum $=1$

If we start with an even sum,
we end up with sum $=0$

The parity of the sum is
an invariant
↳ something that does not change

Game #2.

We have chameleons of
3 different colors red green
blue



When two meet, they both change to the third color

Question: Can we end up with all chameleons of the same color?

starting configuration: (4, 2, 5) ✓

(4, 5, 6)

(1, 3, 6) ✓

+ make up your own

(4, 2, 5) $\longrightarrow$

$\underline{+3}$

(4, 2, 5)

(3, 4, 4)

(5, 3, 3)

(7, 2, 2)

(9, 1, 1)

(11, 0, 0) ✓

(4, 5, 6)

(6, 4, 5)

(8, 3, 4)

(10, 2, 3)

(12, 1, 2)

(14, 0, 1)

(13, 2, 0)

?

(15, 0, 0)

(0, 15, 0) ?

(0, 0, 15)

+3

(1, 3, 6)

(3, 2, 5)

(2, 4, 4)

(10, 0, 0) ✓

(9, 3, 3) (12, 3, 0) (12, 0, 3)

(11, 2, 2)

(13, 1, 1)

(15, 0, 0)

$(n, n+3, m)$ ✓

$\downarrow{+2} \quad \downarrow{-1} \quad \downarrow{-1} \qquad (m \geq 1)$

$(n+2, n+2, m-1)$ ✓

$(n, n+6, m) \qquad (m \geq 2)$

$\downarrow{+2} \quad \downarrow{-1} \quad \downarrow{-1}$

$(n+2, n+5, m-1)$

$\downarrow{+2} \quad \downarrow{-1} \quad \downarrow{-1}$

$(n+4, n+4, m-2)$ ✓

- If two numbers have a difference that is a multiple of 3; it seems to work
- What if this is not the case?

ex. $(\overbrace{4, 5, 6}^{2})$

$(\quad 0, 0)$

$(a, b, c) \qquad a - b$

$(a-1, b-1, c+2) \qquad (a-1)-(b-1) = a-b$

or $(a-1, b+2, c-1) \qquad (a-1)-(b+2) = a-b-3$

or $(a+2, b-1, c-1) \qquad (a+2)-(b-1) = a-b+3$

$-$ changes each time by a multiple of 3.

if $-$ starts as a multiple of 3, it will always be a multiple of 3.

if it starts as not a multiple of 3, it will never be a multiple of 3.

So if we start with (a, b, c) st. $a-b$, $a-c$, $b-c$ are not multiples of 3, then we cannot end up with only one color.

ex. $(4, 5, 6)$ ✗

whether $a-b$ is a multiple of 3 is an invariant of this game.

Extra problem: show that if a difference is a multiple of 3, then it works.

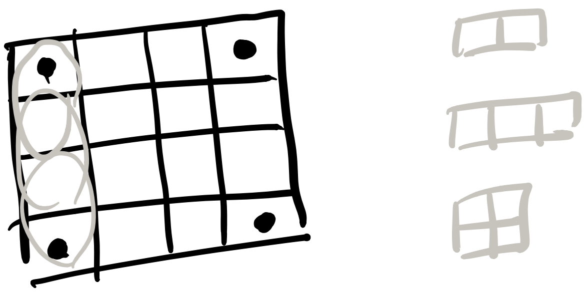
$$\overset{+6}{(7, 1, 0)}$$

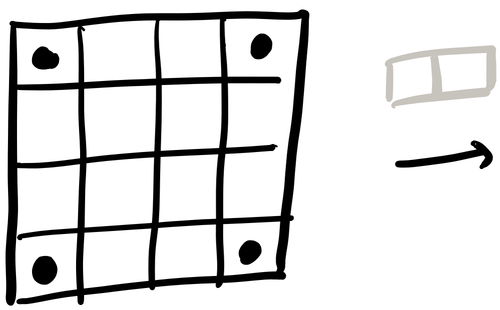
$(6, 0, 2)$

$(5, 2, 1)$

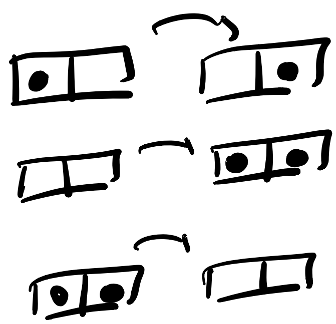
(4, 4, 0)

Game #3 Stomp





→ not possible



# of gophers stay the same

+2

−2

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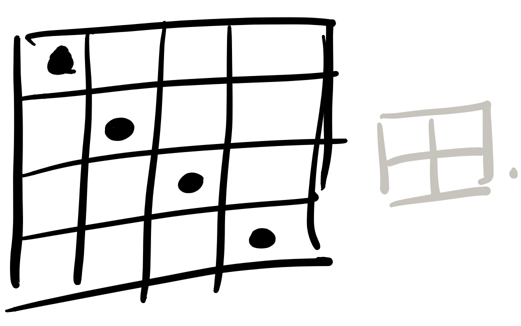
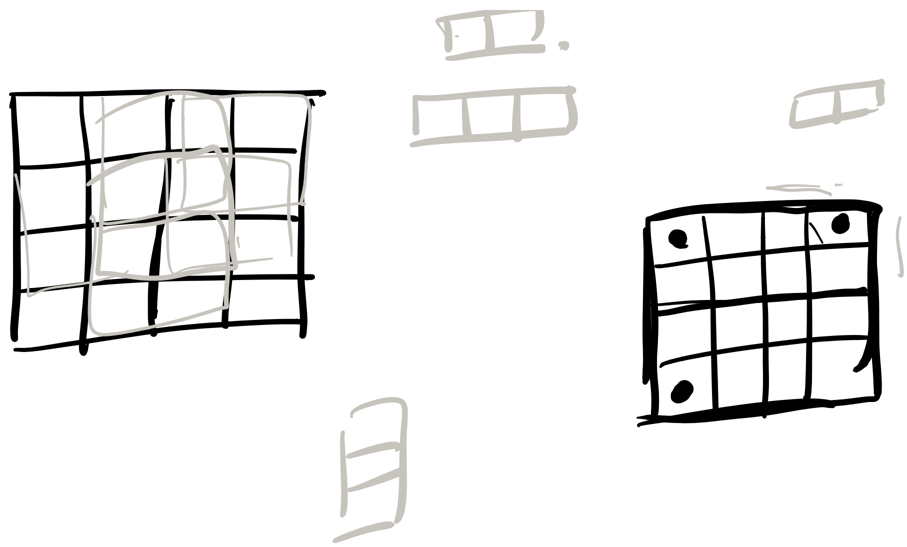
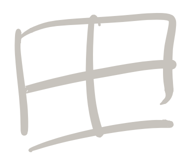
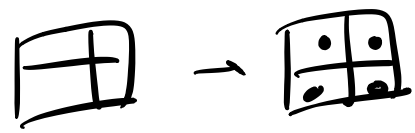
parity of the # of gophers is invariant.

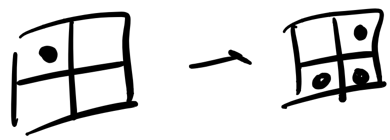




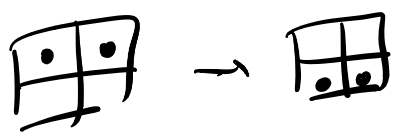




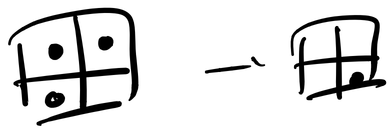
+4



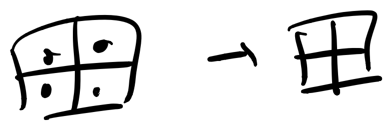
+2



+0



−2

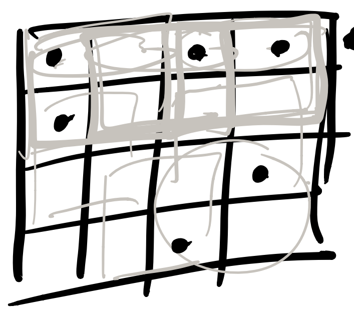


−4

parity is an invariant.

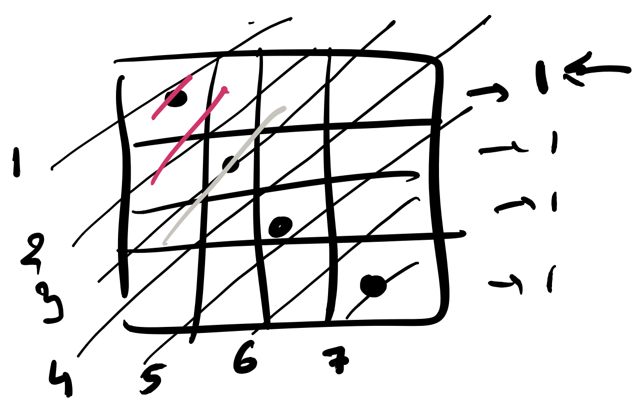
+1

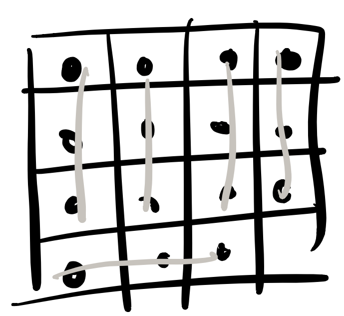
parity is an invariant if the stamp has size a multiple of two.



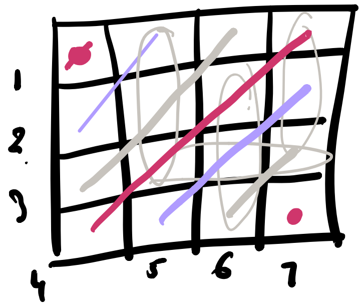
# of sophus changes by either −2, +2, +0

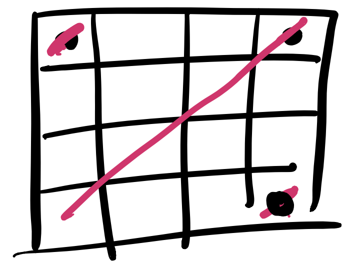
the parity of the # of sophus on the top line is an invariant.





2 red





3	0	0
odd	even	even
even	odd	odd
odd	even	even