Berkeley Math Circle: February 24, 2021 **Bases Part I**

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1. If $N = \overline{abccba}_{59}$, where $a = 1, b = 5, c = 10$, how many factors does N have?

2. If in an unknown base system, $3x^2 - 25x + 66 = 0$, has solutions $x = 4$ and $x = 9$, what is the base?

3. 121_b is a square of an integer for

(A) $b = 10$ only (B) $b = 10$ and $b = 5$ only (C) $2 \leq b \leq 10$ (D) $b > 2$ (E) no value of b

4. $N = \overline{abc}_7 = \overline{cba}_9$. What is b?

5. A base-10 3 digit number n is selected at random. Which of the following is closest to the probability that the base-9 and base-11 representation of n are both 3-digit numbers?

(A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6 (E) 0.7

6. A and B have decided the strategy of their trick beforehand. B leaves the room and C arbitrarily places 64 identical pennies on an 8X8 board, some with heads and some with tails up. C then points to one of the pennies and A flips exactly one penny after doing some calculations. B enters the room and guesses which coin C had pointed. What is the strategy?

7. The first 2007 positive integers are written in base-3. How many of these numbers are base-3 palindromes?

(A) 100 (B) 101 (C) 102 (D) 103 (E) 104

8. In some unknown base, $\overline{ab} + \overline{ab} = \overline{ada}$ for different digits a, b, d. What base is this?

9. Base-8 representation of a perfect square is $\overline{ab3c}$, where $a \neq 0$. Then c equals:

(A) 0 (B) 1 (C) 3 (D) 4 (E) not uniquely determined

10. 121212121212_3 = what number in base 9?