Invariants II: Stomp

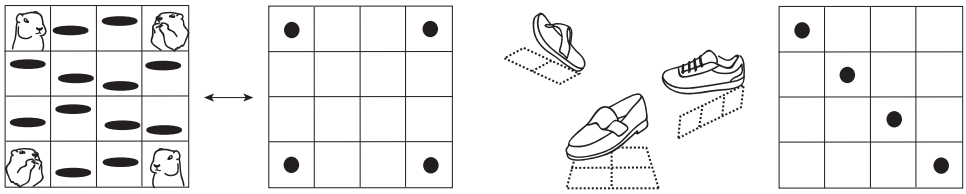
BMC Beg Fall 2021

December 8, 2021

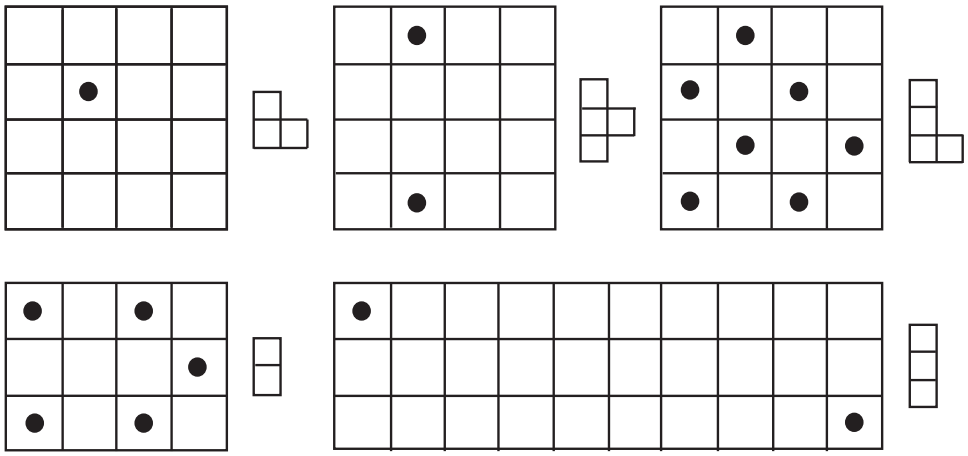
1 Stomp

In the game Stomp (also known as Lights-Out), you are given a Stomp-piece and allowed to place it anywhere within a grid on each move, as long as it lines up with the grid and stays within the boundaries. In the grid, some squares have dots in them. All squares covered by the Stomp piece change state so that dots covered disappear while empty squares that are covered gain a new dot. The goal is to remove all the dots from the grid, or prove that it is impossible to do so.

1. For each of the following puzzles, try to solve the board using a 1×2 domino piece, a 2×2 tetromino square, and finally a 3×1 tromino.



2. For each of the following Stomp boards, find a way to clear the board or prove that it cannot be done.

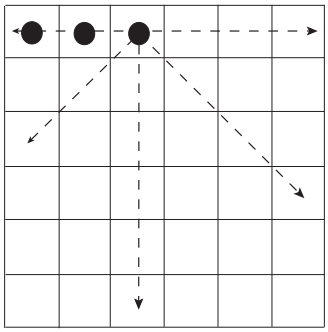


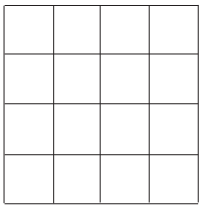
3. Using a T-tetromino on a 3×3 board with a single dot in the top middle square, show how it is possible to end up with a single dot in the central square.

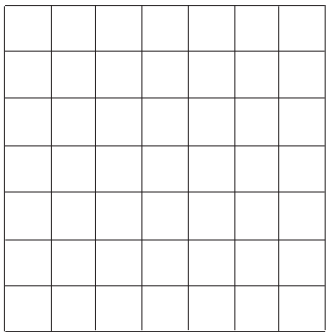
4. Using a T-tetromino on a 3×3 board with a single dot in the top middle square, show how it is possible to end up with a single dot in the bottom middle square.

5. Using a T-tetromino, explain why every Stomp puzzle on an infinite board with an even number of dots is solvable. Why do we need the board to be infinite? Are there Stomp puzzles on a finite board size with an even number of dots that are not solvable?

6. Now imagine a variant where we can reverse the state of all the squares in a single row, column, or diagonal. Can we clear the following board of dots?







7. Find all subsets S whose parity of dots is invariant in the Gopher Gun problem and find all attainable configurations of dots.