Warm Up

- What patterns do you notice?
- Can you write or describe a rule to find the next number in the list?
- Can you write a rule to find any number?

\[0 \ 2 \ 3 \ 4 \ 5\] Raman

\[a_n = 2, 4, 6, 8, \ldots\]

\[\Rightarrow \Rightarrow \Rightarrow \quad \text{even #'s}\]

\[\uparrow \quad \text{Asob } n\]

\[a_n = 2n\]

\[c_n = 3, -5, 7, -9, \ldots\]

\[\Rightarrow \Rightarrow \Rightarrow \quad +2(-1) \cdot 2(-1)\]

"Add 2, alternate between 1, -1"
\[ d_n = 1, 4, 9, 16, \ldots \]

Square numbers

\[ d_n = (n)^2 \]

\[ f_n = 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots \]

divide by 2, divide by 2, divide by 2

numerator is always 3

denominator power 0 of 2

power incremented by 1

\[ f_n = \frac{3}{2^{n-1}} \]

\[ h_n = 1, 1, 2, 3, 5, 8, \ldots \]
Sequences

Def: A sequence is an ordered list of numbers.

\[ a_n = 2, 4, 6, 8, \ldots \]

Each term could be finite or infinite:

\[ a_n = \{a_1, a_2, a_3, \ldots, a_n\} \]

\[ a_n = \{a_1, a_2, a_3, \ldots\} \]

Name of the sequence.
Sequence is like a set, but a set doesn't have to be ordered. The same value can appear many times in a sequence.

\[ a_n = 20, 1, 0, 1, 0, 1, \ldots \]

A set sequence is not a linear sequence.

4 Different types:
- Arithmetic
- Geometric
- Harmonic
- Special

Arithmetic Sequences

Linear sequence:
- Each new term comes by adding to the previous term
- \( d \) = common difference
\( a_n, b_n \)

\[ a_n = \{10, 13, 16, 19, \ldots \} \]

\( +3 \quad +3 \quad +3 \)

\[ d = a_{n+1} - a_n \quad a_1 = 10 \]

recursive rule

next term from the previous

\[ a_{n+1} = a_n + 3; \quad a_1 = 10 \]

genearly:

\[ a_{n+1} = a_n + d; \quad a_1 \]

explicit rule

any term in the sequence.
\[ f(n) = 3(n-1) + 10 \]

\[ f(n) = 3n + 7 \]

generally:

\[ f(n) = a(n-1) + f(1) \]

ex  Find the recursive and explicit rule:

\[
A_0 = 25, 23, 21, 19, \ldots
\]

\[ a = -2 \]
\[ d = 2 \]

\[ f(n) = 25 - 2(n-1) \]

\[ A_{n+1} = A_n - 2; \quad a_1 = 25 \]

recursive

explicit rule
\[ f(n) = 25 - 2n \quad f(n) = 25 - 2(n-1) \]
\[
\begin{array}{ll}
  n = 1 & n = 1 \\
  n = 2 & n = 2
\end{array}
\]

\[ f(n) = 25 - 2(n-1) \]
\[ f(n) = 25 - 2n + 2 \]
\[ f(n) = 27 - 2n \]

\[ b_n = \frac{3}{2} - 4, -1, 2, \ldots 3 \]
\[ b_1 = -4 \quad d = 3 \]

\[ f(n) = 3(n-1) - 4 \]

\[ b_{n+1} = b_n + 3; \quad b_1 = -4 \]

\[ c_n = \frac{3}{2}, 10, 15, \ldots 3 \]

\[ f(n) = 5(n) \]
\[ C_{n+1} = C_n + 5; \quad C_1 = 5 \]

**Geometric sequences**

- Next term is the previous term multiplied by a common ratio.
- \( r = \text{common ratio} \)

- \( a_n = 2, 6, 18, 54, \ldots \)
- \( r = 3 \)

Recursive rule:

- \( a_{n+1} = 3 \cdot a_n; \quad a_1 = 2 \)

Generally:
$$\begin{align*}
\text{An+1} &= r \cdot \text{An} ; \quad a_0 \\
\text{exp} &\text{licitly} \\
\text{f(n)} &= 2 \cdot (3^{n-1}) \\
\text{generally:} \\
\text{f(n)} &= a_0 \cdot (r^{n-1}) \\
\text{f(n)} &= a_0 \cdot \frac{r^n}{r} \\
\text{sequences can either}
\end{align*}$$
Converge  \quad \text{diverge}

Slowly approaching a number  \quad \text{gets very large (\(\infty\) large)}

\[ \lim_{n \to \infty} a_n \rightarrow \text{explicit rule} \]

"limit as \( n \) approaches infinity"

\text{ex.} \quad a_n = \frac{3}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots

\quad a_n = \frac{1}{n}

\[ \lim_{n \to \infty} \frac{1}{n} \quad n = 10 \]

\quad n = 100
\[ n = 10,000,000 \]

\[ a_{10} = \frac{1}{10} = 0.1 \]

\[ a_{100} = \frac{1}{100} = 0.01 \]

\[ a_{10,000,000} = 0.0000001 \]

Our sequence \( a_n \) converges to 0.
example \[ b_n = 2, 4, 6, 8, 10 \]

\[ b_n = 2 \cdot n \]

\[ \lim_{n \to \infty} 2n \]

\[ n = 10 \]

\[ n = 100 \]

\[ n = 10,000,000 \]

sequence always get bigger,

Our sequence \[ b_n \] diverges.