Warm Up

- what patterns do you notice?
- Can you write or describe a rule to find the next number in the list?
- Can you write a rule to find any number?

Raman

① ② ③ ④ ⑤

$a_n = 2, 4, 6, 8, \ldots$

+2 +2 +2

• even #'s

"a sub n"

$a_n = 2 \cdot n$

$c_n = 3, -5, 7, -9, \ldots$

$+2(\cdot -1)$ $-2(\cdot -1)$

"Add 2, alternate between 1, -1"

$d_n = 1, 4, 9, 16, \ldots$

$+3 \quad +5 \quad +7$

square numbrs

$$d_n = (n)^2$$

$f_n = 3, \ 3/2, \ 3/4, \ 3/8, \ \ldots$

$\div 2 \quad \div 2 \quad \div 2$

numerator is always 3
denominator power of 2,
power incremented by 1

$$f_n = \frac{3}{2^{n-1}}$$

$h_n = 1, 1, 2, 3, 5, 8, \ldots$

Sequences

Def a sequence is an ordered list of numbers

$$a_n = \overset{1}{2},\ \overset{2}{4},\ \overset{3}{6},\ \overset{4}{8},\ \dots$$

term

- could be finite, or infinite

$$a_n = \{a_1,\ a_2,\ a_3,\ \dots,\ a_n\}$$

$$a_n = \{a_1,\ a_2,\ a_3,\ \dots\}$$

name of the sequence

Sequence is like a set, but a set doesn't have to be ordered
same value can appear many times in a sequence!

$a_n = \{0, 1, 0, 1, 0, 1, \ldots\}$ $\{0, 1\}$

sequence set

4 Different types:

Arithmetic, Geometric, Harmonic, Special.

$\boxed{\text{Arithmetic Sequences}}$ linear sequences

↳ each new term comes by adding to the previous term

d = common difference

a_n , b_n

ex $a_n = \{10, 13, 16, 19, \ldots\}$

$+3 \quad +3 \quad +3$

$d = a_{n+1} - a_n$

$a_1 = 10$

recursive rule

next term from the previous

$a_{n+1} = a_n + 3 ; \quad a_1 = 10$

generally:

$$\boxed{a_{n+1} = a_n + d ; \quad a_1}$$

explicit rule

any term in the sequence.

* $f(n) = 3(n-1) + 10$

$f(n) = 3n+7$

generally:

$$\boxed{f(n) = d(n-1) + f(1)}$$

$\rightarrow a_1$

ex Find the recursive & explicit rule:

$a_n = \{\overset{①}{25}, \overset{②}{23}, \overset{③}{21}, \overset{④}{19}, \dots\}$

$-2 \quad -2 \quad -2$

$d = -2$

$a_1 = 25$

$f(n) = 25 - 2(n-1)$

explicit rule

$a_{n+1} = a_n - 2;\ a_1 = 25$

recursive

$f(n) = 25 - 2n$

$n = 1$
$n = 2$

$f(n) = 25 - 2(n-1)$

$n = 1$
$n = 2$

$f(n) = 25 - 2(n-1)$

$f(n) = 25 - 2n + 2$

$f(n) = 27 - 2n$

$b_n = \{-4, -1, 2, \dots\}$

$-4 \xrightarrow{+3} -1 \xrightarrow{+3} 2$

$b_1 = -4$

$d = 3$

$f(n) = 3(n-1) - 4$

$b_{n+1} = b_n + 3;\quad b_1 = -4$

$c_n = \{5, 10, 15, \dots\}$

$f(n) = 5(n)$

$C_{n+1} = C_n + 5$; $C_1 = 5$

Geometric Sequences exponential sequence

next term is the previous term multiplied by a common ratio

r = common ratio

ex $a_n = \{2, 6, 18, 54, \dots\}$

$\cdot 3$ $\cdot 3$ $\cdot 3$

$r = \dfrac{a_{n+1}}{a_n}$

$a_1 = 2$
$r = 3$

recursive rule

$a_{n+1} = 3 \cdot a_n$; $a_1 = 2$

generally:

$$\boxed{a_{n+1} = r \cdot a_n \,;\; a_1}$$

explicit rule

$$f(n) = 2 \cdot (3^{n-1})$$

generally:

$$\boxed{f(n) = a_1 \cdot (r^{n-1})}$$

$$f(n) = a_1 \cdot \frac{r^n}{r}$$

Sequences can either

Converge	diverge
slowly approaching a number	gets very (∞) large

$$\lim_{n \to \infty} a_n$$ → explicit rule

"limit as n approaches infinity"

ex.

$$a_n = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \right\}$$

$$a_n = \frac{1}{n}$$

$$\lim_{n \to \infty} \frac{1}{n}$$

$n = 10$

$n = 100$

$n = 10,000,000$

$a_{10} = \frac{1}{10} = 0.1$

$a_{100} = \frac{1}{100} = 0.01$

$a_{10,000,000} = 0.0000001$

sequence is getting closer and closer to 0.

our sequence a_n converges to 0.

ex

$b_n = 2, 4, 6, 8, 10$

$b_n = 2 \cdot n$

$\lim_{n \to \infty} 2n$

$n = 10$

$n = 100$

$n = 10{,}000{,}000$

sequence always get bigger,

Our sequence b_n diverges.