3/7/21

Recap:

sequences: ordered list of numbers

Arithmetic sequences

d = common difference

ex: {10, 13, 16, 19, ...}

$a_{n+1} = a_n + d$; $a_1 =$

$f(n) = d(n-1) + a_1$

Geometric sequences

r = common ratio

ex: {2, 6, 18, 54, ...}

$a_{n+1} = r \cdot a_n$; $a_1 =$

$f(n) = (a_1) \cdot (r^{n-1})$

- $\lim_{n\to\infty} \frac{1}{n} = 0$

converge

- $\lim_{n\to\infty} 2n = \infty$

diverge

Part I : #1-8

① $a_n = n^2 - 1$ Amol

$a_n = \{0, 3, 8, 15, 24, 35, \ldots\}$

$a_1 = (1)^2 - 1 = 0$

$a_2 = (2)^2 - 1 = 3$

$a_3 = (3)^2 - 1 = 8$

$\vdots$

$a_{100} = (100)^2 - 1 = 9,999$

spencer

② $b_1 = 3 \qquad b_n = b_{n-1} + 2$ for all $n > 1$

$b_n = \{3, 5, 7, 9, 11, 13, \ldots\}$

$b_{100} = 2(99) + 3$

$b_{100} = 201$

Abhnav

③ $a_n = 4n - 3$

$a_{21} = 4(21) - 3$

$$a_{21} = 84 - 3$$

$$a_{21} = 81$$

(4) Shreelakshmi

$$a_n = 4(0.5)^n$$

$$a_6 = 4(0.5)^6$$

$$a_6 = 0.0625$$

(5) Ayush

$$a_n = \left\{\left(\frac{1}{n}\right)^{n-1}\right\}$$

$$a_5 = \left(\frac{1}{5}\right)^{5-1}$$

$$a_5 = \left(\frac{1}{5}\right)^4$$

... (5)

$a_5 = \frac{1}{625}$

⑥ a) $\left\{\frac{1}{2^n}\right\}$ Amal

$a_n = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right\}$

$\lim_{n\to\infty} \frac{1}{2^n} = 0$ $\frac{1}{2048}$

this sequence converges to 0.

b) $\{2n-3\}$ Spencer

$\lim 2n-3 = \infty$

$n \to \infty$

this sequence diverges

c) Diverges

d) Converge to 0

e) Diverges

Edward

(7) a) $\{3, 8, 11, 14, \ldots\}$

$a_n = 3n + 2$

$a_n = a_{n-1} + 3; \quad a_1 = 5$

$a_{21} = 3(21)+2$

$a_{21} = 63+2$

$a_{21} = 65$

b) $\{100, 94, 88, 82\}$ Spencer

$-6 \quad -6 \quad -6$

$f(n) = 100 - 6(n-1)$

$a_{15} = f(15) = 100 - 6(15-1)$

$= 100 - 6(14)$

$= 100 - 64 = \boxed{36}$

$= 100 - 84 = \boxed{16}$

(8) $2, \frac{3}{2}, \frac{9}{8}, \frac{27}{32}, \ldots$ Luca Edward

$$r = \frac{3/2}{2} = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

$$a_{n+1} = a_n \cdot \frac{3}{4} \; ; \quad a_1 = 2$$

$$f(n) = 2 \cdot \left(\frac{3}{4}\right)^{n-1}$$

(11) $a_n = n^2 - 5n + 2$ Luca

$n = 2$?

$\mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$

~~$n = 1$?~~ $n = 5$?

$a_n = \{-2, -4, -4, -2, 2\}$

$a_1 = (1)^2 - 5 + 2$

$= 1 - 5 + 2 = -2$

$a_2 = 2^2 - 5(2) + 2$

$= 4 - 10 + 2$

$= -6 + 2 = -4$

$a_3 = -6 + 2$

$= -4$

$a_4 = -2$

$a_5 = +2$

Harmonic sequences

↳ if the reciprocals of all elements form an arithmetic sequence.

reciprocal

$\frac{a}{b} \xrightarrow{\text{reciprocal}} \frac{b}{a}$

$\{1, 1/2, 1/3, 1/4, \ldots\}$

$\{1, 2, 3, 4, 5, \ldots$

$+1 \quad +1 \quad +1$

$\{1, 1/2, 1/4, 1/8, \ldots\}$

$\{1, 2, 4, 8, \ldots\}$