Warm-Up

$a_n = 2, 4, 6, 8, \ldots$

$+2 \quad +2 \quad +2$

multiples of 2

add 2 each time

$a_n = 2(n)$

$a_n = 1, 4, 9, 16, \ldots$

$+3 \quad +5 \quad +7 \quad +9$

square

perfect squares

$a_n = (n)^2$

$$f_n = 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots \frac{3}{16}$$

1, 2, 3, 4, 5

$\cdot \frac{1}{2}$ $\cdot \frac{1}{2}$ $\cdot \frac{1}{2}$

$$f_n = \frac{3}{2}\left(\frac{1}{2}^n\right) \qquad f_n = \frac{3}{2n}$$

$$f_n = \frac{3}{2^{n-1}}$$

$$h_n = 1, 1, 2, 3, 5, 8, \ldots$$

$2 = 1 + 1$ $\quad$ $5 = 2 + 3$

Sequences

Def a sequence is an ordered list of numbers

each item = term. organized

by term #"

• sequences can be finite or infinite

position; term #'s

$$a_n = \{a_1, a_2, a_3, \ldots a_k\}$$

name of the sequence

terms

$$a_n = \{a_1, a_2, a_3, \ldots\}$$

Sequence is like a set...

$\{0, 1, 0, 1, 0, 1\}$ $\{0, 1\}$

ordered #'s repeat no order no #'s repeat sequence   set

if $a_1, a_2, a_3, \ldots, a_n$ is a sequence then the corresponding sum of terms,

$$S = a_1 + a_2 + a_3 + \cdots + a_n$$

is called a <u>series</u>.

can also be infinite or finite

4 types of sequences:
Arithmetic, geometric, harmonic, and special (fibonacci, triangular, etc)

Arithmetic sequences

linear sequences

next term has a common difference w/the previous term.

d = common difference

ex $a_n = \{ 10, 13, 16, 19, \ldots \}$

+3 +3 +3

$d = +3$ $a_1 = 10$

recursive rule

next term from the previous

$$a_{n+1} = a_n + 3 ; \quad a_1 = 10$$

generally:

$$\boxed{a_{n+1} = a_n + d}$$

explicit rule

give us any term in the sequence

$$f(n) = 3(n-1) + 10$$

$$f(n) = 3n + 7$$

generally:

$$\boxed{f(n) = d(n-1) + a_1}$$

ex: Find the explicit and recursive rule for the following sequences:

$$\boxed{f(n) = d(n-1) + a_1} \quad \boxed{a_{n+1} = a_n + d}$$

$$a_n = \{25, 23, 21, \ldots\}$$

$$-2 \quad -2$$

$$d = -2$$

$$a_1 = 25$$

$$a_{n+1} = a_n - 2; \; a_1 = 25$$

$$f(n) = -2n + 27$$

$$f(n) = -2(n-1) + 25$$

1

$$b_n = \{-4, -1, 2, \ldots\}$$

$$+3 \quad +3 \quad +3$$

$$f(n) = 3(n-1) - 4$$

$$b_{n+1} = b_n + 3; \; b_1 = -4$$

$C_n = \{5, 10, 15, \dots\}$

$+5 \quad +5$

$C_1 = 5$

$C_{n+1} = C_n + 5 \; ; \; C_1 = 5$

$f(n) = 5(n-1) + 5$

$f(n) = 5n$

$\boxed{\text{Geometric sequences}}$ exponential sequences

Current term, times a common ratio gives you the next term

r = common ratio

$\underline{ex}$ $\quad a_n = \{\overset{2\cdot 3^0}{2}, \overset{2\cdot 3^1}{6}, \overset{2\cdot 3^2}{18}, 54, \ldots\}$

$1 \quad \cdot 3 \quad 2 \quad \cdot 3 \quad 3 \quad \cdot 3 \quad 4$

$a_1 = 2 \qquad r = 3$

$r = \dfrac{a_{n+1}}{a_n}$

recursive rule

$a_{n+1} = 3 \cdot a_n \;; \quad a_1 = 2$

generally

$$\boxed{a_{n+1} = r \cdot a_n \;; \quad a_1}$$

explicit rule

$$f(n) = 2(3^{n-1}) \qquad f(n) = \frac{2}{3} \cdot 3^n$$

generaly:

$$\boxed{f(n) = a_1 (r^{n-1})}$$

ex. $a_n = \{3, 3/2, 3/4, 3/8, \ldots\}$

$$a_{n+1} = \frac{1}{2} \cdot a_n \; ; \quad a_1 = 3$$

$$f(n) = 3\left(\left(\frac{1}{2}\right)^{n-1}\right)$$

Sequences can either <u>converge</u>

or diverge

infinitely large.

closer and closer to a value

$$\lim_{n \to \infty} a_n$$ explicit rule

"The limit as n approaches ∞"

$a_n = 2, 4, 6, 8, 10, \ldots$

$a_n = 2(n-1) + 2$

$a_n = 2(n)$

$n = 10$

$n = 100$

$n = 10,000,000$

$$\lim_{n\to\infty} 2(n-1)+2 \rightarrow \infty$$

the sequence diverges.

$$b_n = \underset{①}{1},\ \underset{②}{1/2},\ \underset{③}{1/3},\ 1/4,\ \ldots$$

$$b_n = \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{1}{n}$$

$n = 10$ $\quad \frac{1}{10} = 0.1$

$n = 100$ $\quad \frac{1}{100} = 0.01$

$n = 10{,}000{,}000 \quad \left\{ \quad \frac{1}{10{,}000{,}000} = 0.0000001 \right.$

our sequence is converging to 0.