Warm-Up

0

$\frac{2}{+2} \frac{4}{+2} \frac{6}{+2} \frac{8}{+2}$

Multiples of 2

Add 2 each time

$an = 2(n)$

$1, 4, 9, 16, ...$

Square

Perfect squares

$an = (n)^2$
\[ f_n = \frac{3}{2} \left( \frac{1}{2} \right)^n \]

\[ h_n = 1, 1, 2, 3, 5, 8, \ldots \]

\[ 2 = 1 + 1 \quad 5 = 2 + 3 \]

**Sequences**

**Def** a sequence is an ordered list of numbers

each item = term, organized
by term #, 

• sequences can be finite or infinite

\[ a_n = 2a_1, a_2, a_3, \ldots a_n \]

↑

name of terms

the sequence

\[ a_n = 3a_1, a_2, a_3, \ldots 3 \]

sequence is like a set...

\[ \{0, 1, 0, 1, 0, 1, 3\} \]

↑

no #’s repeat

ordered

↑

#’s repeat

↑

no order

no #’s repeat
if $a_1, a_2, a_3, \ldots, a_n$ is a sequence then the corresponding sum of terms,

$$S = a_1 + a_2 + a_3 + \ldots + a_n$$

is called a series. It can also be infinite or finite.

4 types of sequences:
- Arithmetic
- Geometric
- Harmonic
- Special (Fibonacci, triangular, etc.)
Arithmetic sequences

Next term has a common difference with the previous term.

d = common difference

Ex: \( a_n = 10, 13, 16, 19, \ldots \)

\[ \begin{align*}
3 & \quad 3 & \quad 3 \\
\end{align*} \]

\( d = +3 \quad a_1 = 10 \)

Recursive rule

Next term from the previous

\( a_{n+1} = a_n + 3; \ a_1 = 10 \)
generally:

\[ a_{n+1} = a_n + d \]

**explicit rule**
gives us any term in the sequence.

\[ f(n) = 3(n-1) + 10 \]
\[ f(n) = 3n + 7 \]

generally:

\[ f(n) = d(n-1) + a_1 \]

**ex:** Find the explicit and recursive rule for the following sequences:
\[ f(n) = d(n-1)^2 + a_1 \]
\[ a_{n+1} = a_n + d \]

\[ a_n = 25, 23, 21, \ldots 3 \]
\[ d = -2 \]
\[ a_1 = 25 \]

\[ a_{n+1} = a_n - 2; \quad a_1 = 25 \]

\[ f(n) = -2n + 27 \]
\[ f(n) = -2(n-1) + 25 \]

\[ b_n = 3, -4, -1, 2, \ldots 3 \]

\[ f(n) = 3(n-1) - 4 \]

\[ b_{n+1} = b_n + 3; \quad b_1 = -4 \]
\[ C_n = \frac{3}{5}, 10, 15, \ldots ; \quad C_1 = 5 \] 

\[ C_{n+1} = C_n + 5 ; \quad C_1 = 5 \]

\[ f(n) = 5(n - 1) + 5 \]

\[ f(n) = 5n \]

Geometric Sequences

Current term, times a common ratio gives you the next term

f = Common ratio
Example: $a_n = \frac{2}{3^n} \cdot 2^{3n}$

$a_1 = 2$, $r = 3$

Recursive rule:

$r = \frac{a_{n+1}}{a_n}$

Explicit rule:

$a_{n+1} = r \cdot a_n$; $a_1 = 2$
\[ f(n) = 2 \left( 3^{n-1} \right) \]

\[ f(n) = \frac{2}{3} \cdot 3^n \]

generally:

\[ f(n) = a_1 \cdot (r^{n-1}) \]

\[ a_n = \frac{3}{2}, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots \]

\[ a_{n+1} = \frac{1}{2} \cdot a_n \; ; \; a_1 = 3 \]

\[ f(n) = 3 \left( \frac{1}{2} \right)^{n-1} \]

sequences can either converge
or diverge

\[ \text{closer and} \]
\[ \text{closer to a value} \]

\[ \lim_{n \to \infty} a_n \]

"The limit as \( n \) approaches \( \infty \)"

\[ a_n = 2, 4, 6, 8, 10, \ldots \]

\[ a_n = 2(n-1) + 2 \]

\[ a_n = 2n \]

\[ n = 10 \]

\[ n = 100 \]
\[ n = 10,000,000 \]

\[ \lim_{{n \to \infty}} 2(n-1)+2 \to \infty \]

The sequence diverges.

\[ b_n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \]

\[ b_n = \frac{1}{n} \]

\[ \lim_{{n \to \infty}} \frac{1}{n} \]

\[
\begin{array}{c}
\text{n = 10} \\
\text{n = 100}
\end{array}
\]

\[
\begin{array}{c}
\frac{1}{10} = 0.1 \\
\frac{1}{100} = 0.01
\end{array}
\]
\[ n = 10,000,000 \ \Rightarrow \ \frac{1}{10,000,000} = 0.0000001 \]

Our sequence is converging to 0.