Sequence: an ordered list of numbers

Arithmetic Sequences

ex: $\{10, 13, 16, 19, \ldots\}$

$+3 \quad +3 \quad +3$

d = common difference

$$a_{n+1} = a_n + d \; ; \; a_1$$

$$f(n) = d(n-1) + a_1$$

Geometric Sequences

ex: $\{2, 6, 18, 54, \ldots\}$

$\cdot 3 \quad \cdot 3 \quad \cdot 3$

r = common ratio

$$r = \frac{a_{n+1}}{a_n}$$

$$a_{n+1} = r \cdot a_n \; ; \; a_1$$ ← recursive

$$f(n) = a_1 (r^{n-1})$$ ← explicit

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

converges to 0

$$\lim_{n \to \infty} 2n = \infty$$

diverges.

Part I

Dona

① $a_n = (n)^2 - 1$

$a_n = \{0, 3, 8, 15, 24, 35, \ldots\}$

$a_1 = (1)^2 - 1$

$= 0$

$a_{100} = 9,999$

② $b_1 = 3 \quad b_n = b_{n-1} + 2$ Aargav

$b_n = \{3, 5, 7, 9, 11, 13, \ldots\}$

Daniel

$b_{100} = 2(99) + 3$

$b_{100} = 201$

③ $a_n = 4n - 3$ Mansi

$a_1 = 4(1) - 3 = 1$

$a_{21} = 4(21) - 3$

$a_{21} = 81$

part I #41-10

(4) $a_n = 4(0.5)^n$ Shavryen

$a_6 = 4(0.5)^6$

$a_6 = \frac{1}{16} = 0.0625$

(5) $a_n = \left(\frac{1}{n}\right)^{n-1}$ Shyla

$a_5 = \left(\frac{1}{5}\right)^4$

$a_5 = 0.00\,16$

$= \frac{1}{625}$

(6) a) $\left\{\frac{1}{2^n}\right\}$ Ming Yi

$a_n = \{1/2, 1/4, 1/8, \ldots\}$

$\lim_{n \to \infty} \frac{1}{2^n} = 0$

this sequence converges to 0.

b) $\{2n - 33$ Adithi

$b_n = \{-1, 1, 3, \ldots\}$

$\lim_{n \to \infty} 2n - 3 = \infty$

this sequence diverges.

c) $\{2^n\}$ Aaryav

$c_n = \{2, 4, 8, \ldots\}$

$\lim_{n \to \infty} 2^n = \infty$

this sequence diverges.

d) $\{(0.1)^n\}$ Mansi

$d_n = \{0.1, 0.01, 0.001, \ldots\}$

$\lim_{n \to \infty} 0.1^n = 0$

This sequence converges to 0.

e) $\left\{\left(\frac{3}{2}\right)^n\right\}$ Dna

$a_n = \{ 3/2, \ 9/4, \ 27/8, \ \ldots \}$

$\lim_{n \to \infty} \left(\frac{3}{2}\right)^n = \infty$

This sequence diverges.

part I: #7-13

(7) a) 5, 8, 11, 14 Ad ithi

a_{21}

Recursive Rule

$a_{n+1} = a_n + 3 \; ; \; a_1 = 5$

Explicit Rule

$f(n) = 3(n-1) + 5$

$a_{21} = f(21) = 3(21-1) + 5$

$f(21) = 3(20) + 5$

$f(21) = 65$

b) 100, 94, 88, 82,

Ming Yi

$a_{n+1} = a_n - 6 \; ; \; a_1 = 100$

$f(n) = -6(n-1) + 100$

$$a_{15} = f(15) = -6(15-1)+100$$

$$= -84+100$$

$$= 16$$

(8a) $2, 3/2, 9/8, \ldots$

$r = \frac{3/2}{2} = \frac{3}{4}$ Η κοινή διαφορά

$$a_{n+1} = \frac{3}{4}(a_n); \quad a_1 = 2$$

$$f(n) = 2\left(\left(\frac{3}{4}\right)^{n-1}\right)$$

$$a_{11} = 2\left(\frac{3^{10}}{4^{10}}\right)$$

Aiden

b) $-0.3,\ 0.6,\ -1.2,\ 2.4$

$$f(n) = (-0.3)\left((-2)^{n-1}\right)$$

$$a_{n+1} = -2(a_n);\quad a_1 = -0.3$$

$$a_{20} = (-0.3)\left((-2)^{19}\right)$$

Adithi

9 a) $a_1 = 3;\quad a_n = a_{n-1} - 7$

$$a_n = \{3, -4, -11, -18, -25\}$$

$a_1 = 3$

$a_2 = -4$

$a_3 = a_2 - 7 = -11$

$a_4 = a_3 - 7 = -18$

$a_5 = a_4 - 7 = a_5 = -25$

Neenash

b) $b_1 = 4 \qquad b_n = 2 \cdot b_{n-1} - 1$

$b_2 = 2(b_1) - 1$

$= 8 - 1 = 7$

$b_3 = 7(2) - 1$

$= 13$

$b_4 = 2(13) - 1$

$= 25$

$b_5 = 2(25) - 1$

$= 49$

$b_n = \{4, 7, 13, 25, 49\}$

Harmonic sequence

↳ reciprocal of the elements form an arithmetic sequence.

ex $\{1, 1/2, 1/3, 1/4\} = \{\frac{1}{n}\}$

$\{1, 2, 3, 4, 5, \ldots\}$

(10) a) 1, 1, 2, 3, 5, 8 ooha

"phi" = ϕ ratio consecutive terms

$$a_{n+1} = a_n + a_{n-1}; \; a_1 = 1$$

$$a_2 = 1$$

b) $a_{n+1} = n(a_n); \; a_1 = 1$