BMC Intermediate II
Nov. 10, 2021

Induction or Not? Part I

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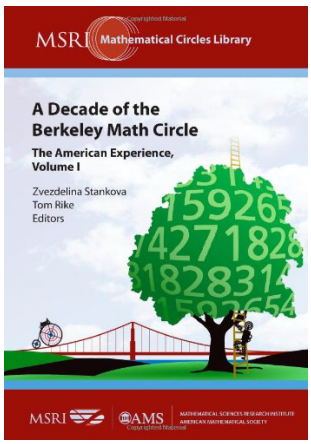
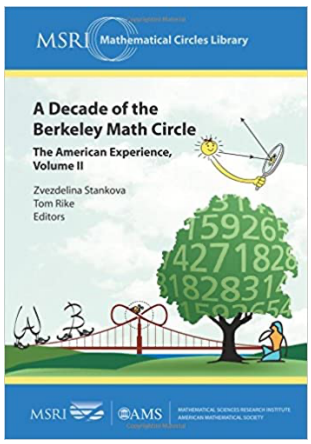
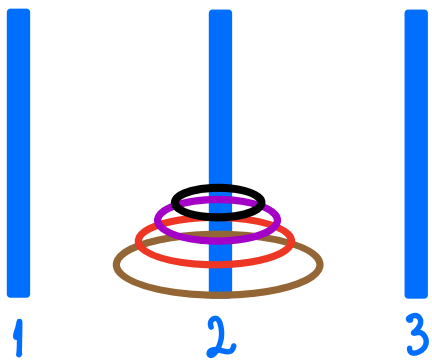
BMC-Upper Director





① Towers of Hanoi:



Recurrence formula:

$$\begin{cases} a_n = 2a_{n-1} + 1 & \forall n \geq 2 \\ a_1 = 1 \end{cases}$$

Direct formula:

$$a_n = 2^n - 1 \qquad \forall n \geq 1$$

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② To be or not to be MMI?

(a) (Euler) $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{n^2} + \cdots = \frac{\pi^2}{6}$

(b) (Riemann) All non-trivial zeros of $\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$ have real part $= \frac{1}{2}$.

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③ Summations by MMI or not?

(a) $1+2+3+\cdots+n=\frac{n(n+1)}{2}$

(b) $1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$

(c) $1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2$

(d) $1+3+5+\cdots+2n-1=n^2$

(e) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{(n-1)\cdot n}=1-\frac{1}{n}$

(f) $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\cdots\left(1-\frac{1}{n^2}\right)=?$

(g) $1+2+2^2+2^3+\cdots+2^n=?$

(h) $1+2\cdot 2+3\cdot 2^2+4\cdot 2^3+\cdots+n\cdot 2^{n-1}=?$

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④ ∫-Afficionados: (For Calculus die-hards only!)

Prove by MMI:

$$\int_0^1 x^n\cdot e^x\,dx=(-1)^{n+1}n!+e\sum_{k=0}^{n}(-1)^{n-k}\frac{n!}{k!}$$

Hint: Use ∫ by parts.

⑤ Creating Change

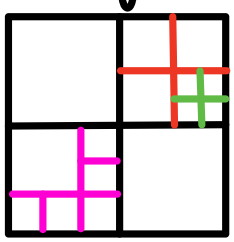
- $3, $5 bills
- What sums can we pay?

n	1	2	3	4	5	6	7	8	9	10	11	12	13
Y/N													

⑥ Squares

Into how many squares can we cut a square?

n	1	2	3	4	5	6	7	8	9	10	11	12	13
Y/N	✓	✗	✗	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓



Prove: this is impossible for $n=2,3,5$.