EXPECTED VALUE AND GAMES OF CHANCE

Ocean games!

In all of the games below, there is a octopus who likes to recruit young mathematicians to play his games, plus one player (you can imagine it's you). Our currency will be sand dollars; assume all of the sand dollars have the same value.

1. In this first game, the octopus has a fair coin, with a red fish on one side, and a blue fish on the other. To play, the octopus flips the coin – if it comes up red, you win 1 sand dollar, and if it comes up blue, you win 2 sand dollars. What is the fair price of a ticket for this game?

2. Next, we'll change it up a bit. Suppose the octopus still has a fair coin, with a red fish on one side, and a blue fish on the other. However, this time you have to choose a color before the flip. If it comes up red AND you chose red, you win 1 sand dollar. If it comes up blue AND you chose blue, you win 2 sand dollars. If you chose the wrong color, you don't win a prize. Is one of the colors a better bet? What's a fair price for a ticket?

3. For the next game, the octopus has 3 tokens, equal in shape/weight/feel/etc. You can only tell them apart by looking at them. Suppose two of the tokens have moray eel stickers on them, and one has a sea lion sticker. The octopus puts all three tokens into his top hat. Then you 1) choose an animal and 2) draw one token without looking. If there's a match, you win! The prizes are 4 sand dollars for a sea lion and 2 sand dollars for a moray eel. Is it better to choose sea lion or moray eel? What's the fair price for a ticket?

4. The octopus has a fair 4-sided die (*what shape is that?*) – one side has a penguin, one side has a green turtle, one side has a pink anemone, and one side has a tiger shark. The octopus rolls the die. No matter what, you win a prize! The prize values are as follows:

	penguin	turtle	anemone	shark
prize	18 sd	3 sd	5 sd	2sd

   If the price of a ticket for this game is 6 sand dollars, is the game more likely to be profitable for the octopus, or for you? If you could afford to play 2021 times in a row, would you do it? What if you could only play a few times? (Assume you love sand dollars very much.)

5. The octopus has a deck of ten cards – half have spiny lobsters on them and half have moon jellies. The octopus will shuffle and then deal 3 cards to you. If you are dealt exactly N lobster cards, then you will win 2^N sand dollars. What is the fair price of this game? (Hint: permutations and combinations might be helpful here!)

6. Final game, and this one is tough! The octopus has six treasure chests, and each one has a different (integer) number of sand dollars inside, from 1 to 6. They are all mixed up, and you don't know which one is which. If you buy a ticket to this game, you can choose one treasure box and open it to see how many sand dollars are inside. Then you have a choice – you can keep that treasure chest OR you can set it aside, pay one extra sand dollar, and choose a different chest to keep. If you swap, you keep the second box. When should you swap and when should you keep the first box?

   What is the fair price of a ticket for this game?

   Variation: How does your answer change if, when you swap, you must put your treasure chest back in the pile and let the octopus mix them all up again before choosing a second box?

HOMEWORK: Test yourself to see if you understood the ideas from today.

7. (Similar to Game 1.) The octopus has a fair coin, with a red fish on one side, and a blue fish on the other. To play, the octopus flips the coin – if it comes up red, you win 11 sand dollars, and if it comes up blue, you win 3 sand dollars. What is the fair price of a ticket for this game?

8. (Similar to Game 2.) Suppose the octopus still has a fair coin, with a red fish on one side, and a blue fish on the other. However, this time you have to choose a color before the flip. If it comes up red AND you chose red, you win 11 sand dollars. If it comes up blue AND you chose blue, you win 3 sand dollars. If you chose the wrong color, you don't win a prize. Is one of the colors a better bet? What's a fair price for a ticket?

9. (Similar to Game 3.) This time the octopus has 7 tokens, equal in shape/feel/etc. You can only tell them apart by looking at them. Suppose four of the tokens have barnacle stickers on them, two of the tokens have pilot whale stickers on them, and one has a vampire squid sticker. The octopus puts all seven tokens into his top hat. Then you choose an animal and draw one token without looking. If there's a match, you win! If the prizes are 2 sand dollars for a barnacle, 4 sand dollars for a pilot whale, and 6 sand dollars for a vampire squid, which animal should you choose, and what is a fair price for the ticket?

   Can you modify **one** of the prizes to ensure that all three animals are equally good choices? Then what is the fair price of a ticket?