Berkeley Math Circle
Trigonometry and the Unit Circle

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April 14th and April 21st

Warm-Up

Write down everything you know about right triangles.

TRIGONOMETRY AND THE UNIT CIRCLE

1. For ΔABC, with side $a = 20$, $c = 21$, find side b, $cos(C)$, angle C, and angle A.

2. For ΔABC, with side $a = 30$, $c = 16$, find side b, $sin(A)$, $cos(A)$, angle C, and angle A.

3. For ΔABC, with side $a = 40$, $c = 9$ find side b, find $sin(A)$, $cos(A)$, $cos(C)$, $sin(C)$ and compare your answers. What conclusions can you make?

4. For ΔABC, with side $a = 30$, $c = 40$, find side b, $sin(A)$, $cos(A)$ and $tan(A)$, angle C, and angle A.

5. Find the length of the side labeled x.
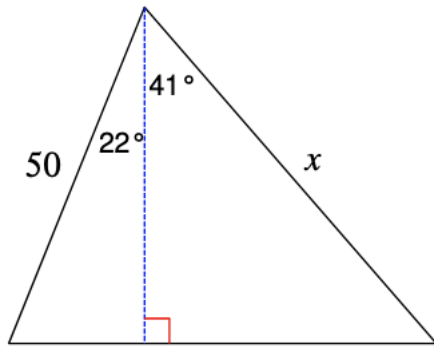
(a)
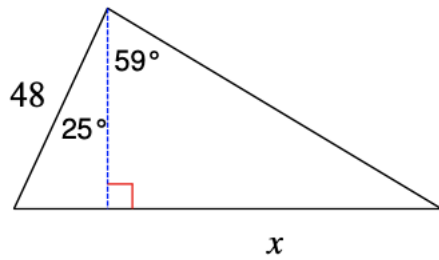
(b)

6. Convert the following degrees to radians.

A) 120°   **B)** 210°   **C)** −60°

E) −110°   **F)** 330°   **G)** −45°

I) 300°   **J)** −135°   **K)** 450°

7. Convert the following radians to degrees.

A) $\dfrac{\pi}{6}$

B) $\dfrac{5\pi}{3}$

C) $-\dfrac{\pi}{2}$

D) $\dfrac{3\pi}{4}$

E) $-\dfrac{\pi}{4}$

F) $-\dfrac{5\pi}{6}$

G) $\dfrac{7\pi}{6}$

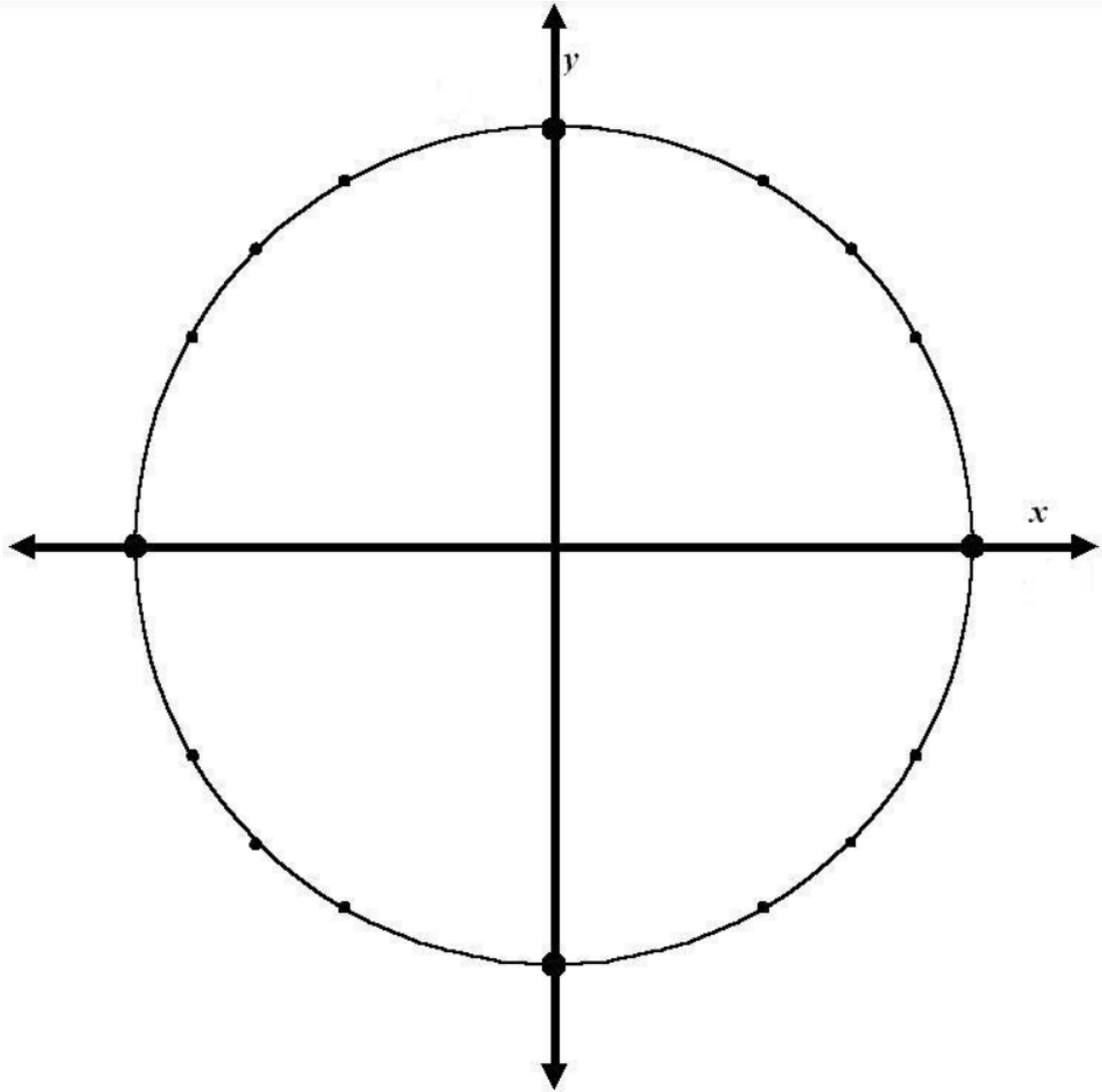**H)** $-\dfrac{\pi}{6}$
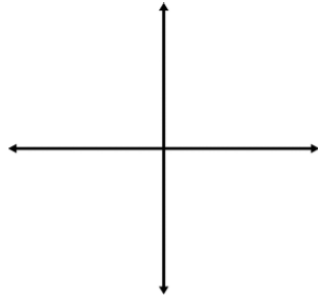
I) 2.3

J) $\dfrac{11\pi}{6}$

K) -1.28

L) $-\dfrac{2\pi}{3}$

8. Fill in the unit circle. Each position should have the degree measurement, the radian measurement, $(cos(\theta), sin(\theta))$
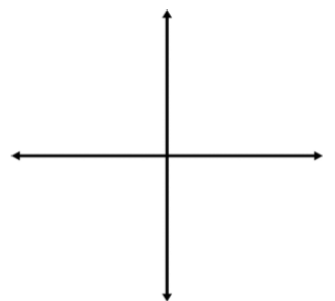
9. Sketch each angle in standard position, and find the a negative and positive cotermi-
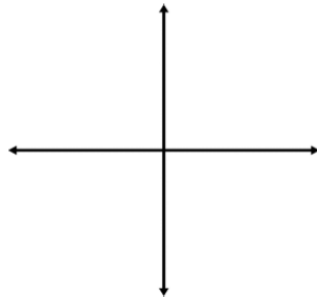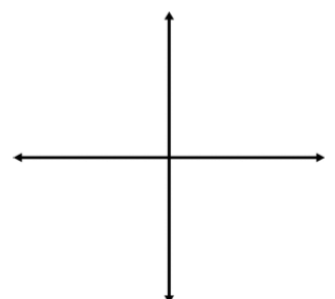nal angle, then find the *sin* and *cos* of the given angle.
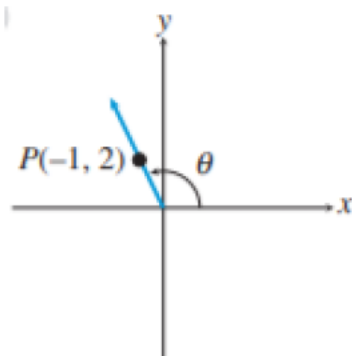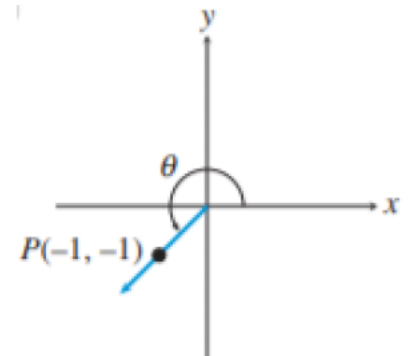
a) 300°

b) −120°

c) $\dfrac{5\pi}{4}$

d) $-\dfrac{11\pi}{12}$

10. Draw a reference triangle and evaluate the six trigonometric functions of θ.

a)

$P(-1, 2)$ θ

b)

θ

$P(-1, -1)$

11. The point P is on the terminal side of angle θ. Sketch a reference triangle. Evaluate the six trigonometric functions for θ. Write "undefined" if the trig function has no value.

    a) $P(4,3)$ b) $P(22,-22)$
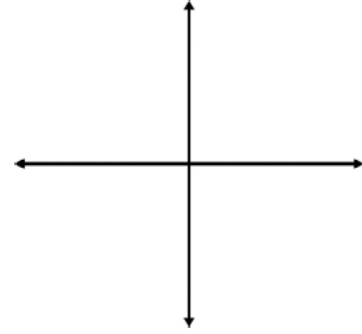
    c) $P(0,-4)$ d) $P(-2,0)$

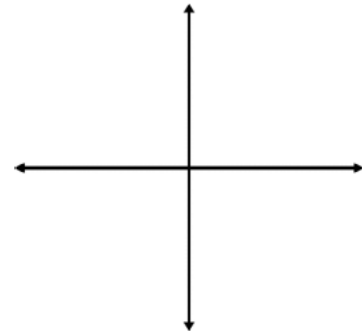12. Sketch a reference triangle in the appropriate quadrant for the given info.

a) Given: $\cos\theta = \dfrac{2}{3}$, $\cot\theta > 0$
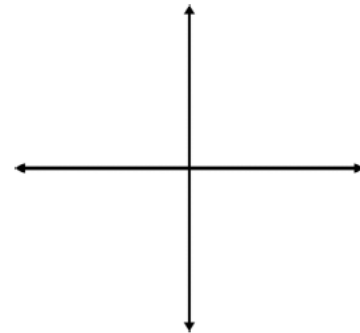
 Find $\sin\theta$ and $\tan\theta$

b) Given: $\sin\theta = \dfrac{1}{4}$, $\tan\theta < 0$

 Find $\cos\theta$ and $\cot\theta$
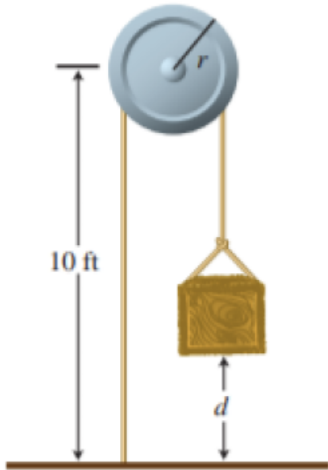
c) Given: $\sin\theta = \dfrac{-2}{5}$, $\cos\theta > 0$

 Find $\tan\theta$ and $\sec\theta$

13. A simple pulley with radius 4in used to lift heavy objects is positioned 10 feet above ground level. If the pulley rotates at 720 degrees, determine the height, d, to which the object is lifted.

14. The inside lane 1 of a running track has an inner radius of 33 meters. Each lane has a 1 meter width.

    (a) Find the distance around the curve in lane 1.
   (b) Find the distance around the curve in lane 8.

15. A bike's wheels are 28 inches in diameter. For high gear the pedal sprocket is 9 inches in diameter and the wheel sprocket is 3 inches in diameter. At racing speed of 66 ft / sec in high gear.
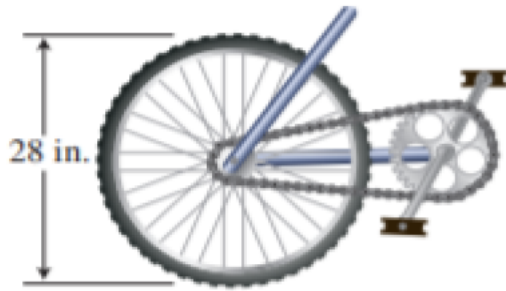
(a) Find the angular velocity of the wheel in radians per second

(b) Find the angular velocity of the wheel sprocket in radians per second

(c) Find the angular velocity of the pedal sprocket in revolutions per minute

16. The radius of Earth is approximately 3963.2 miles. It takes 23h 56m 4.1s for the Earth to rotate once on its axis.

(a) Find the angular velocity of the earth's rotation in radians per hour.

(b) Find the linear velocity on the Equator

(c) Where would the linear velocity equal zero?

17. The moon rotates once on its axis in approximately 27 days and rotates once around the earth in approximately 27 days. What effect does this have on our view of the moon?

18. The angle of elevation of the top of a tree is 30 from a point 28 ft away from the foot of the tree. Draw and label a diagram, and find the height of the tree rounded to the nearest foot.

19. A ladder with its foot on a horizontal flat surface rests against a wall. It makes an angle of 70 degrees with the horizontal. The foot of the ladder is 4 ft from the base of the wall.

    (a) Draw and label a sketch of the situation.
    (b) Find the height of the point where the ladder touches the wall.
    (c) Find the length of the ladder.

20. From the top of the Empire State building1250ft , the angle of depression to a car on ground level is 36 degrees. Draw a diagram and find the distance from the base of the building to the car.

21. A passenger in a plane passenger in an airplane sees two towns directly to the left of the plane. Find d, x and y as shown in the diagram.
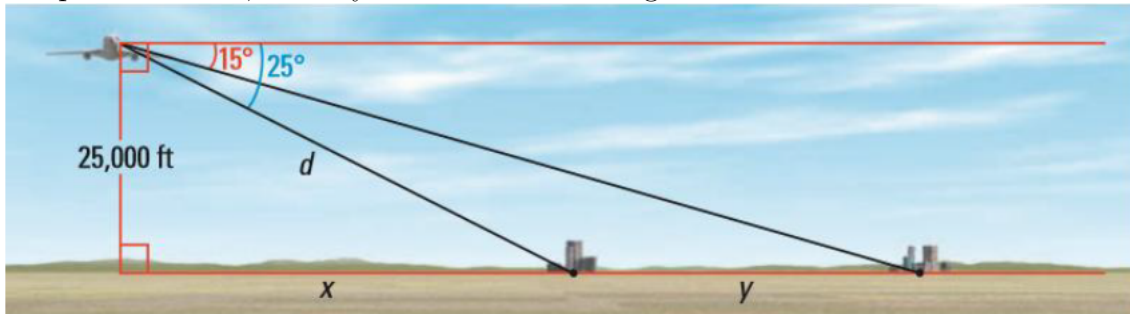
22. A man on the deck of a ship is 15 ft above sea level. He observes that the angle of elevation of the top of a cliff is 70 degrees and the angle of depression of its base at sea level is 50 degrees.

   (a) Sketch and label a diagram
   (b) Find the height of the cliff
   (c) Find the distance from the ship to the base of the cliff .