Suppose that a fair coin is tossed ten times. We can record the outcome of the event as a string of ten *H*'s and *T*'s, understanding that the probability of an *H* or *T* occurring at any particular position is ½.

1. If this experiment were conducted twenty times, then in how many cases would one expect the sequence of ten *H*'s and *T*'s to begin with a *T*?

2. Flipping a coin ten times gives how many possible outcomes?

3. For how many outcomes will there be exactly three heads? Consequently, what is the probability of obtaining three heads?

4. If this experiment were conducted twenty times, then in how many cases would one expect to obtain exactly three heads?

5. On the other hand, in how many of the twenty trials do we expect to obtain exactly six heads?

6. Consider all the possible outcomes from tossing a fair coin ten times. In how many cases will the first tail occur on the very first flip? In how many cases will it occur on the second flip?

7. Make a table listing the number of outcomes in which the first occurrence of a tail is on the kth flip, for each *k* in the range $1 \leq k \leq 10$.

8. Based on your table, calculate the average number of flips required to obtain the first tail.

9. Suppose we were to flip a coin repeatedly until it came up tails. What is the expected number of flips needed to accomplish this task?

10. There is a subtle difference in the previous two questions. What is it?

11. Given an unfair coin, how could we empirically determine the probability (presumably not equal to ½) that the coin comes up heads?

12. What does it mean to say that an event will occur with probability 2/3? What about with probability $(\sqrt{2}-1)$?

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1   These materials taken from Sam Vandervelde's *Math Circle in a Box*, Chapter 10: "Heads or Tails."