**Encrypted Arithmetic I**[1]

1. Shmerlin the Magician is expecting several wicked witches to come to his house. Shmerlin heads to the Wizards Shop to purchase dried frogs as gifts for the witches. Every guest is supposed to get the same (nonzero) number of frogs, and Shmerlin does not want to have any leftovers. What is the smallest number of frogs that Shmerlin should buy if he knows that:

a. either 3 or 5 witches are coming?

b. either 4 or 6 witches are coming?

2. a. Joe the Gold Miner has seven golden nuggets. The first nugget is worth $1, the second one $2, the third one $3, and so on. Joe heads for the country store with three of these nuggets in his pocket. He claims that with these three nuggets he will be able to pay any amount expressed as a whole number of dollars, from $1 to $7. Which three nuggets are in Joe's pocket?

b. Now Joe has fifteen nuggets worth 1, 2, … 15 dollars. Which four of these should he take with him if he wants to be able to pay any integer amount ranging from $1 to $15?

3. See board.

4. a. Three purses are filled with gold dust. How can you arrange them in order from heaviest to lightest using 3 weighings on a balance scale with two pans?

b. Four purses are filled with gold dust. How can you arrange them in order from heaviest to lightest using 5 weighings on a balance scale with two pans?

5. A pair of gold coins, a pair of silver coins, and a pair of bronze coins have been placed on the table. In each pair, one coin is counterfeit—lighter than the real coin. All the real coins have the same weight, and the counterfeit coins are all of the same weight as well. How can you find all three counterfeit coins using 2 weighings on a balance scale with two pans?

1 These sessions taken from *Mathematical Circle Diaries, Year I*, Sessions 19 and 20.