Tiling a Plane - Optimal Social Distancing Position

Berkeley Math Circle - Fall 2020
By Harry Main-Luu

Some overarching questions for today's session:

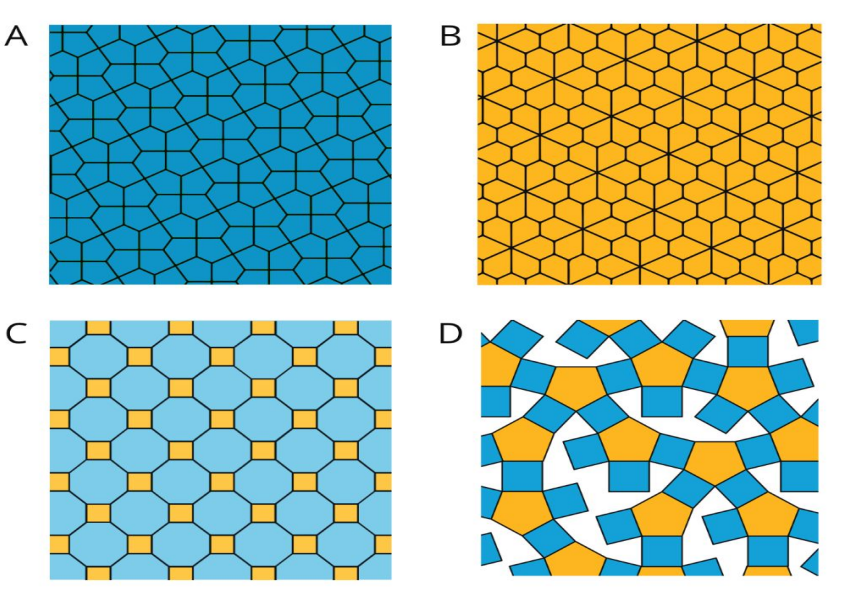
What is the optimal social distancing position and WHY?

Can certain Tetris pieces tile a finite board?

Parts of materials are taken from High School for the Gifted entrance exam, BMC Book Volume 1, and U. Waterloo, Canada.

Tiling a Plane: Warm-Ups

Choose the odd one out!

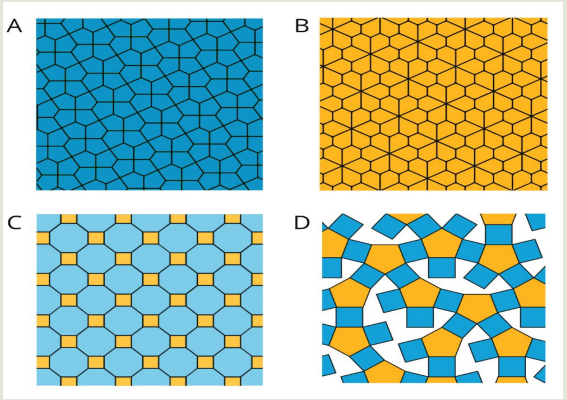


Graphics from open sources

Tiling a Plane: What is it?

Definition: A *tessellation* of the plane is a way to cover the entire plane using **finitely many** types of geometric figures, where there are no overlapping or gaps.

Examples: A, B, C (earlier).
Non-example: D



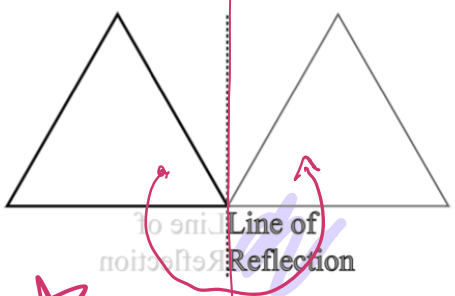
Try to make your own shape that will tile the plane, using up to three different tiles!

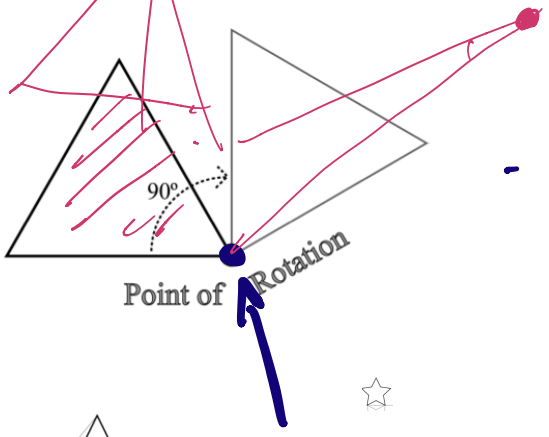
You can of course use any shape. But some strategies might help:

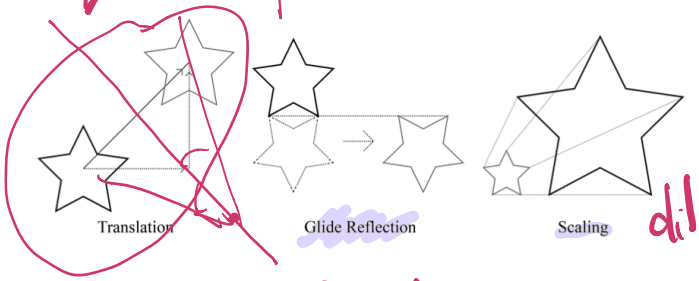
- Use polygons (shapes with straight sides rather than curved)
- Use symmetries (translation, rotation, reflection work best)

Some standard symmetries of the plane.

Reflection





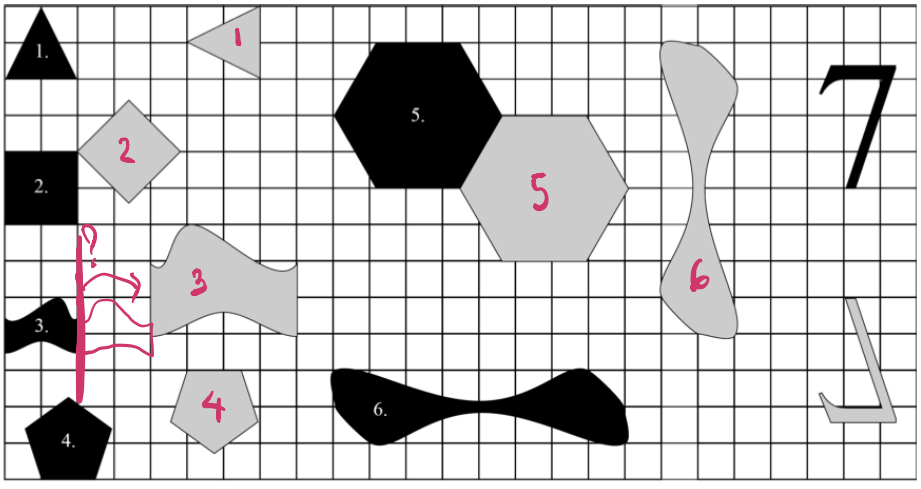


dilation

two shapes in the plane:

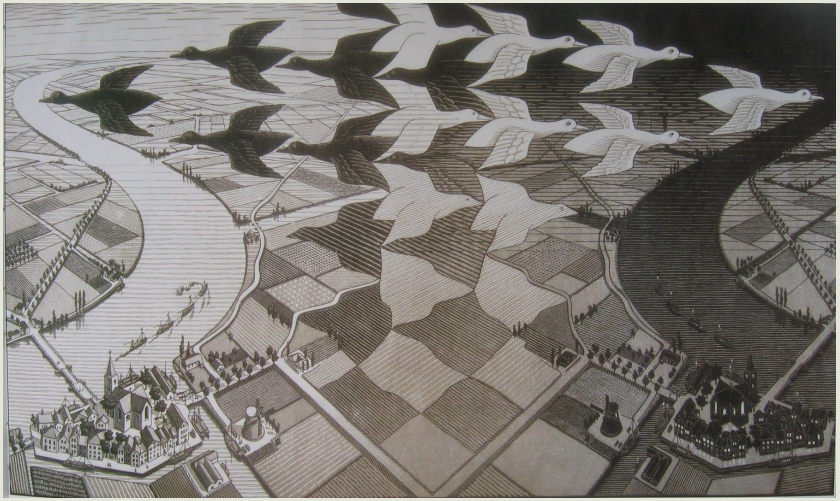
- similar: if they are results from applying above symmetries a few times.
- congruent: if they are results from applying above symmetries a few times, except dilations

Ex: describe the following symmetries:

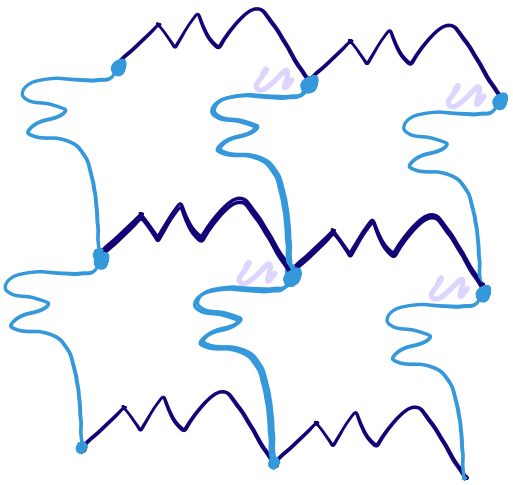


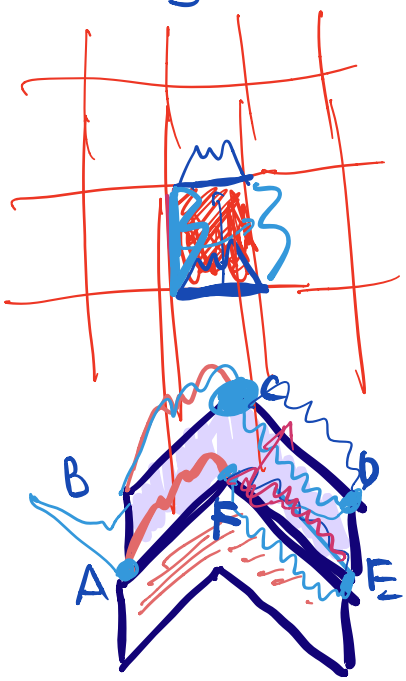
Escher Tessellations

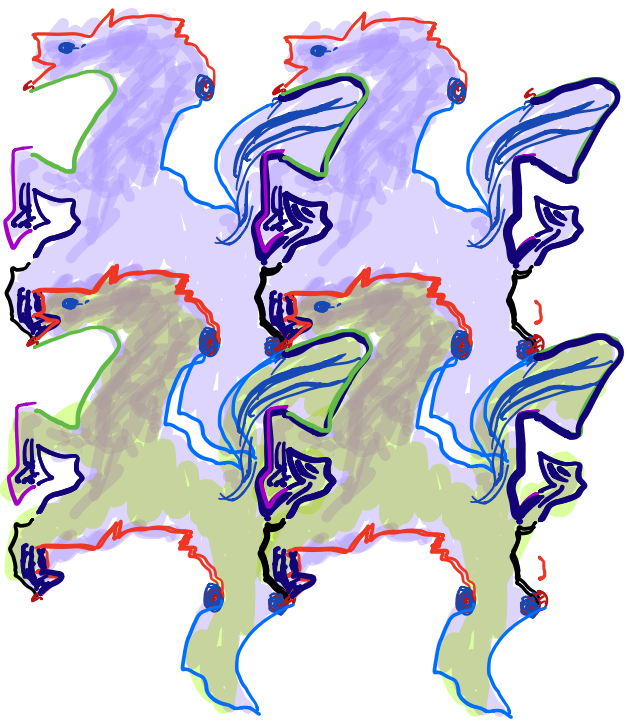
So here is one cool way to make tessellations, due to a Dutch artist M.C Escher. This technique has appeared on tapestries, wallpapers, floor tiles...



- Start from 1 corner → next corner
- copy your line to the "touching" side.



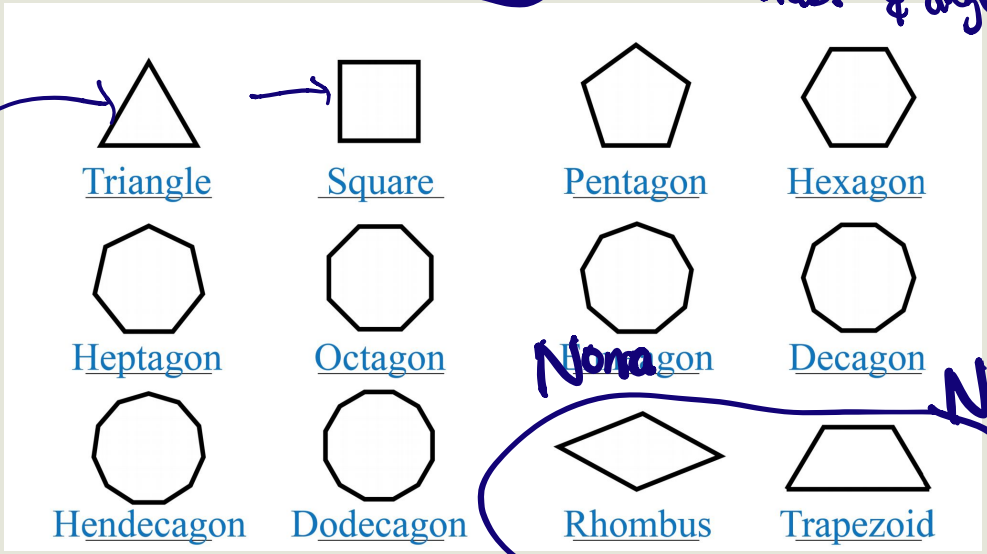




Which special shapes can tile a plane and WHY?

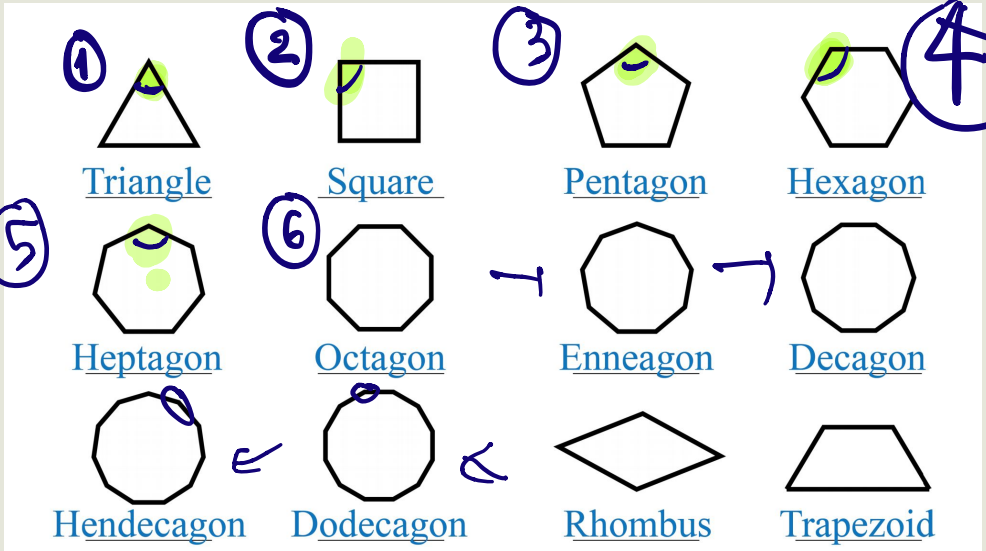
We will focus on regular polygons! What is a regular polygon?

sides & angles are all the same



Which special shapes can tile a plane and WHY?

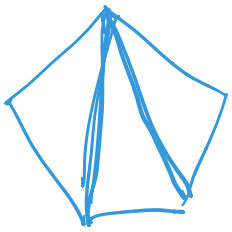
Which regular polygons can tile the plane by itself? Explore and practice!



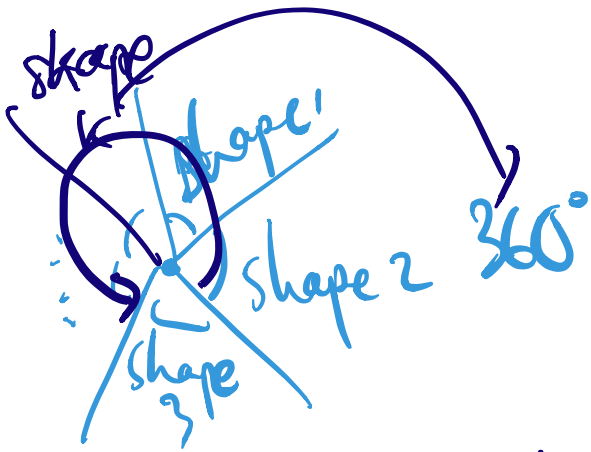
There are ∞- many types of reg. polygons

some YES

some NO



→ angles must add to 360°.



Say if I can tessellate by k many shapes.

then: $\angle \text{shape} * k = 360°$

interior angle of reg. n-gon?

n-gon: total $180° \times (n-2)$?

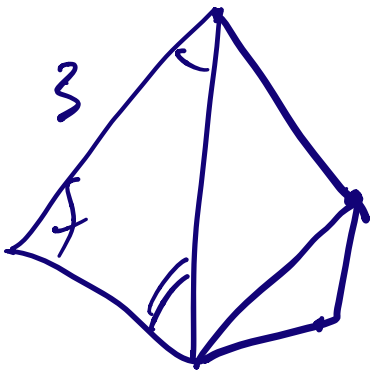
$180° \cdot (n-2)$

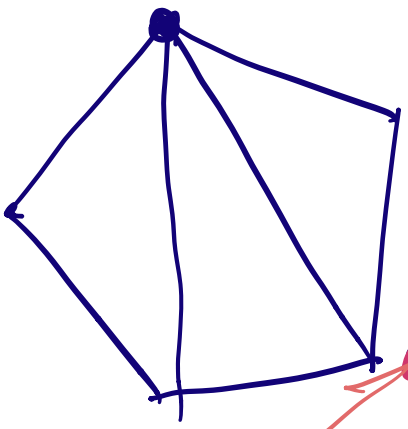
recursive

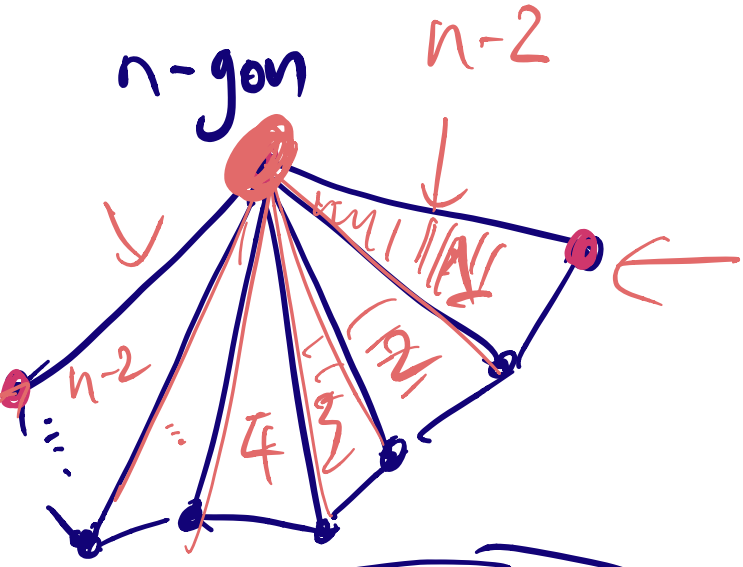
1st Δ

$180° * (\#\text{ add})$

$n-2$

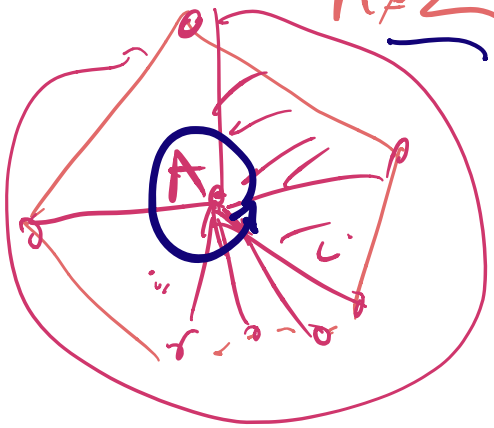






$180°(n-2) = 180° \cdot n - 360°$

$n \neq \triangle - 1 \bigcirc = $ ~~total~~ total K



n points from A

→ nΔ

Sum of all interior angles of a regular n-gon?

Number of Angles	Total Degrees	Measurement of Each Angle
3	180°	$\frac{180}{3} = 60$ ✓
4	? 360°	$\frac{360}{4} = 90$
5	? $540 = 180 \times 3$	
6		
8		
n	$180 * (n-2)$	$\frac{180° * (n-2)}{n}$

$\angle_{shape} * k = 360°$

$$\left(\frac{180° \cdot (n-2)}{n} \cdot k = 360°\right) * n$$

$$\frac{180°}{180}(n-2)\cdot k = \frac{360}{180} n$$

$$(n-2)k = 2n$$

Same

$$(n-2)k - 2n = 0$$

$$(n-2)k - 2(n-2) - 4 = 0$$

$$(n-2)(k-2) = 4 \to 2\cdot 2,\ 1\cdot 4,\ 4\cdot 1$$

$n-2 \geq 1$; $k-2$ natural #.

Solve for n, k at the same time!

but n, k being natural #'s

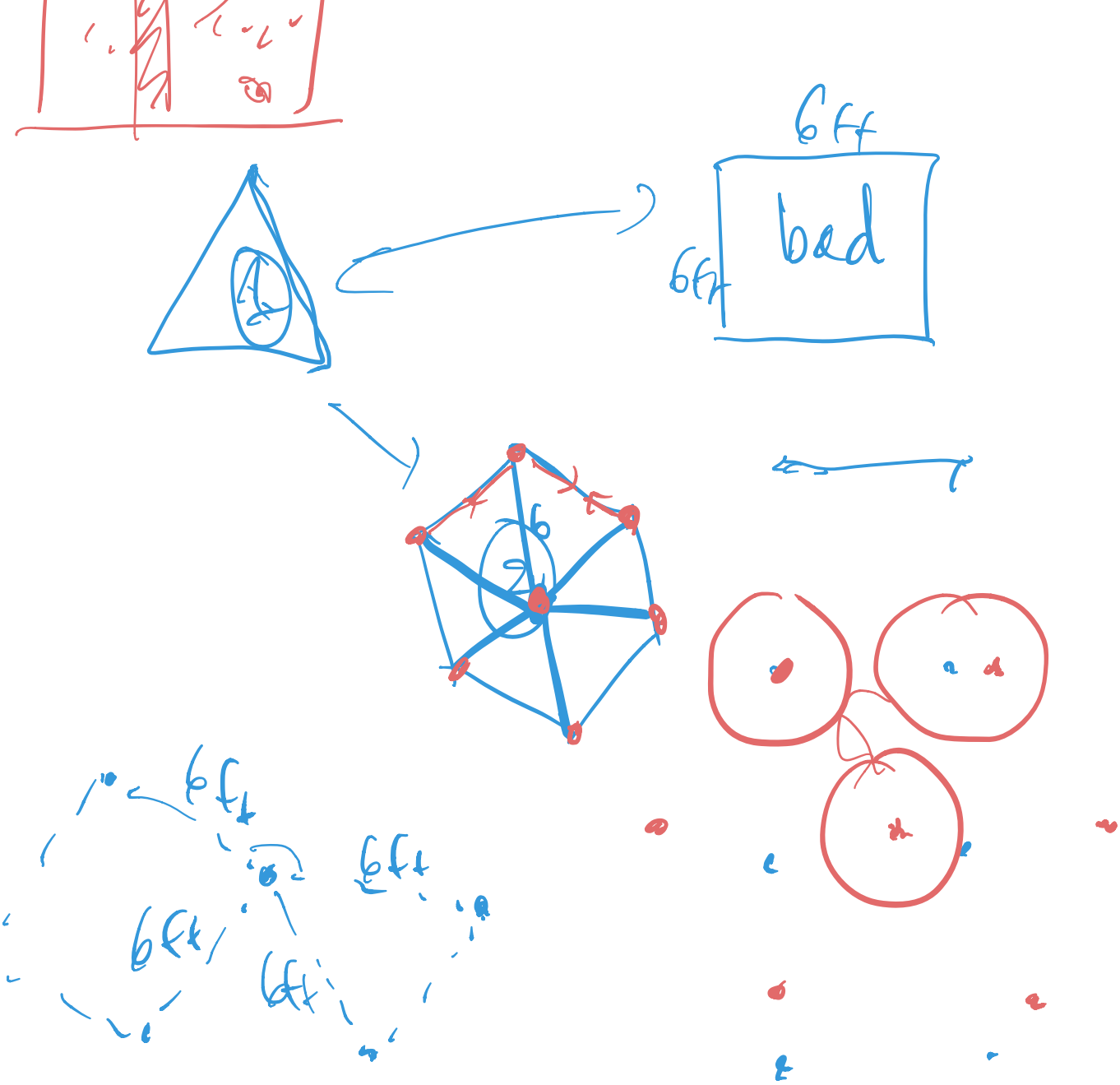
diophatine equation

eq^n.

$n-2 = 2 \to n = 4$

$= 1 \to n = 3$

$= 4 \to n = 6$


bad

Which regular polygons can tile the plane by itself?

Number of Angles	Measurement of Each Angle	Can We Tile?
3	60	YES
4	40	YES
5	72	
6		
8		
n	--	--

Expansion: What about combinations of regular polygons?

1. We want to remodel 1015 Evans and retile the floor using a combination of two types of regular polygons from selection of 3-gon, 4-gon, 6-gon or 8-gon. Which combinations are possible?

2. What if we want to use three different types?