Problems From the Moscow Math Circle[1]

Problem 1. A beaker filled to the brim with water weighs 5 pounds, while the same beaker filled halfway weighs 3.25 pounds. How many pounds of water can the beaker hold?

Problem 2. Which is greater, 333333 x 444444 or 222222 x 666667? By how much?

Problem 3. Given a triangle *ABC* with angle *B* = 90 and *AB* = *BC* = 1, and a point *M* chosen at random on *AC*, is it possible to tell what the sum of the distances from *M* to *AB* and from *M* to *BC* will be?

Problem 4. When Peter broke his piggy bank, it contained no more than 100 coins. He divided the coins into piles of 2 coins each, but was left with one extra coin. The same happened when Peter divided the coins into piles of 3 coins, piles of 4 coins, and piles of 5 coins. Each time he was left with one extra coin. How many coins were in the piggy bank?

Problem 5. A rectangular steel plate measuring 17 x 10 inches has been traced on a piece of paper. Using only the plate, the paper, and a pencil, find the center of the paper rectangle.

Problem 6. Angela has 7 potatoes, Minh has 5, and Greg has none. They combine their potatoes to make a bowl of mashed potatoes and share the bowl of potatoes equally among the three of them. In exchange for his share, Greg gives Minh and Angela 12 pieces of chocolate. How should they divide the chocolate between them, if they are to be fair?

Problem 7. Is it possible to cut several circles out of a square of side 10cm, so that the sum of the diameters of the circles would be 5 meters or more?

CHALLENGE: For an experiment a researcher puts a dot of invisible ink on a piece of paper and also draws a square with regular ink on the paper. In the experiment, a subject will draw a visible straight line on the page and the researcher, who has on special eyeglasses for spotting the dot, will tell the subject which side of the line the dot of invisible ink is on. If the dot is on the line, the researcher will tell the subject it is on the line. What is the smallest number of straight lines the subject needs to draw to figure out for sure whether the invisible dot lies in the square?

CHALLENGE: A train moves in one direction for 5.5 hours. If the train covers any 100-mile segment of the journey in 1 hour: (a) Is the train necessarily moving at a constant rate? (b) Is the train's average speed necessarily 100 mph?

1 These sessions taken from *A Moscow Math Circle: Week-by-Week Problem Sets*, Problem Set 1.