Mathematical Games of Strategy II[1]

Problem 1. A long time ago, when a father was 27 years old, his son was 3. Now the father is three times as old as his son. How old is the son?

Problem 2. The following facts are true about all zoos in NeverLand:
- If a zoo has both hippos and rhinos, then this zoo has no giraffes.
- Every zoo has a hippo or a rhino. (Some zoos have both)
- If a zoo has both a hippo and a giraffe, then this zoo also has a rhino.
- The zoo in the capital of NeverLand has a giraffe. Does this zoo have a rhino? How about a hippo?

Problem 3. A teacher gave each of his first-grade students three marbles. Some of these marbles were yellow, and others were red. First the teacher asked the students with two or more red marbles to raise their hands, and 13 hands were raised. Next, she asked the students with two or more yellow marbles to raise their hands, and 15 hands were raised. Lastly, the students who had two marbles of different color raised their hands, and 17 hands were raised. How many students received three marbles of the same color?

Problem 4. Tim and Alex are playing a game. They have 2 jars of cookies, one with 31 cookies and the other with 27. On every turn, a player is allowed to take up to 6 cookies from the same jar (a player has to take at least 1 cookie per turn). The person who takes the last cookie wins the game. Tim goes first. Who has a winning strategy, and what is it?

Problem 5. Tim and Alex are playing a game. They have a pile of 56 marbles. On a single turn, a player can take up to 8 marbles from the pile. The player who cannot make his move loses the game. Tim goes first. Who has a winning strategy, and what is it?

Problem 6. Tim and Alex are playing a game: they take turns breaking a rectangular chocolate bar into pieces. The bar is 6 squares wide by 10 squares long. On his turn, a player snaps an existing piece of chocolate into two smaller pieces (breaking along the line between squares). The player who cannot take a turn loses the game. Who has a winning strategy, and what is it?

Problem 7. Tim and Alex are playing a game. The numbers 1 through 10 are written on the board in a row. The players take turns inserting plus and minus signs between the numbers. After all nine signs have been placed, the resulting expression is evaluated; that is, the additions and subtractions are performed. The first player wins if the answer is even, the second wins if the answer is odd. Tim goes first. Who has a winning strategy, and what is it?

Problem 8. Twenty soldiers, all of different heights, are standing in line in random order. On your command, any two soldiers that are separated from each other by one soldier can switch places. You are allowed to repeat this command as many times as you want to. Will it always be possible to arrange your soldiers by height?

CHALLENGE: Tim and Alex are playing a game on a board of size 5 x 2011. On his turn, each player crosses out a 2 x 2 square that does not intersect with any square that has already been crossed out. (In this game, two squares intersect if they share at least one 1 x 1 square). The player who cannot make such a move loses the game. Tim goes first. Who has a winning strategy, and what is it?

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1 These sessions taken from *Mathematical Circle Diaries, Year I*, Sessions 27 and 28.