Mathematical Games of Strategy I[1]

Problem 1. Suppose you are playing two chess games on two boards at the same time. Both of your opponents are chess world champions. How should you organize the games and how should you play in order to win or have a draw in at least one of the games?

Problem 2. Tim and Alex are playing a game. They have 2 piles of candy, 23 pieces in each. On every turn, a player is allowed to take up to 5 candies from any single pile. (Skipping turns is not allowed). The winner is the person who takes the last candy. Tim goes first. Who has a winning strategy, and what is it?

Problem 3. Tim and Alex take turns placing bishops on the squares of the 8 x 8 chessboard in such a way that the bishops cannot attack one another (In this game, the color of the bishops is irrelevant). Tim goes first. Who has a winning strategy, and what is it?

Problem 4. Tim and Alex are playing a game. The first player writes a one-digit number on the board. After that, the players take turns adding digits to this number. They stop when the number reaches 16 digits. Tim wins if the resulting 16-digit number is divisible by 9; Alex wins otherwise. Alex goes first. Who has a winning strategy, and what is it?

Problem 5. Tim and Alex take turns placing 1 x 2 dominos on a 10 x 10 chessboard. Each domino should be placed so as to fully cover two squares of the board; the dominoes cannot overlap. The player who cannot place a domino loses. Tim goes first. Who has a winning strategy, and what is it?

Problem 6. Suppose that p and q are two prime numbers.
- How many different factors does p^2 have?
- How many different factors does p^5 have?
- How many different factors does p^x have, where x is some positive integer?
- How many different factors does $p \times q$ have?
- How many different factors does $p^2 \times q$ have?

Problem 7. Two players take turns putting quarters on a round table. The coins are allowed to touch, but they cannot overlap. The players cannot move quarters that have already been placed. The player who cannot place a quarter loses the game. Who has a winning strategy, and what is it?

CHALLENGE: Come up with a number that ends in 17, is divisible by 17, and has the sum of its digits equal to 17.

1 These sessions taken from *Mathematical Circle Diaries, Year I*, Sessions 27 and 28.