The first piece of paper has the label \( C_1 C_2 C_3 \).

The first to fall asleep is \( C_2 \).

\( C_1, C_2, C_3 | M_1, M_2 | U_2, U_3, U_4 \)

\( C_2 C_3, C_1, C_3, C_4 \), etc...

\( C_1 C_2 M_1, C, H_1 C_2 \)...

The box will have 9.8.7 pieces of paper.

The box is called SAMPLE SPACE.

\[ D = A \cup B, \quad A - \text{all three from the same country} \]

\[ B - \text{all three from different countries} \]

\[ A, B \text{ disjoint} \]

\[ P(A \cup B) = P(A) + P(B) \]

\[ P(A) = \frac{|A|}{9.8.7}, \quad \text{we need } |A| = ? \]

\[ |A| = |A_c| + |A_m| + |A_v|, \quad A_c - \text{all three from Canada} \]

\[ A_m - \text{all three from Mexico} \]

\[ A_v - \text{all three from US} \]

\[ |A_m| = 0 \]

\[ |A_v| = 4 \cdot 3 \cdot 2 = 24 \]

\[ |A| = 6 + 24 = 30 \]

\[ P(A \cup B) = P(A) + P(B) = \frac{30}{9.8.7} + \frac{24.6}{9.8.7} = \frac{54.6 + 24.6 \times 3}{9.8.7} \]

\[ = \frac{5 + 38.8 \times 3}{3 \times 4.7} = \frac{5 + 24}{12.7} = \frac{29}{84} \]

\[ \Rightarrow P(D) = 1 - P(D^c) = 1 - \frac{29}{84} = \frac{55}{84} \]

1. 2 Mexico 3 Canada 4 US

3 fell asleep.

What is the probability that exactly two of those who fell asleep are from the same country?

Replace the problem with a new one in which we have a box full of pieces of paper.

Blindfolded person takes a paper from the box and then opens the eyes and reads the content.

\[ S = \{ (x,y,z) : x,y,z \text{ are distinct elements of} \{ C_1, C_2, C_3, M_1, M_2, U_1, U_2, U_3, U_4 \} \} \]

\[ |S| = 9.8.7. \]

\[ P( C_1 M_3 U_1) = \frac{1}{9.8.7} \]

\[ D \subseteq S \]

\[ D \text{ corresponds to the situation in which exactly two of the sleeping ones are from the same country.} \]

\[ P(D) = ? \]

Trick: \( P(D^c) \) is easier to calculate.

\[ |B| = 24 \]

Canadian can be chosen in 3 ways.

U.S. person in 4 ways.

Mexican person in 2 ways. \( 4 \times 3 \times 2 = 24 \) ways.

\[ |B| = 24.6 \times 3 \]

\[ P(B) = \frac{|B|}{9.8.7} = \frac{24.6 \times 3}{9.8.7} \]

\[ = \frac{3 \times 4.7}{12.7} = \frac{15}{29} = \frac{24}{84} \]

\[ \Rightarrow P(D) = 1 - P(D^c) = 1 - \frac{24}{84} = \frac{55}{84} \]
New solution:

\[ T = \{ c_1, c_2, c_3 \} \]

\[ c_1 c_2 c_3 \] is illegal way to write

\[ c_3 c_2 c_1 \] illegal way to write

\[ M_1 C_3 U_1 \] illegal way to write

\[ C_3 M_1 U_1 \]

\[ T = \{ (x_1 y_1 z_1), (x_2 y_2 z_2) \text{ different elements from} \{ c_1, c_2, c_3, M_1, M_2, U_1, U_2, U_3, U_4 \} \]

and \((x_1 y_1 z_1)\) is in lexicographic order?

\[ |T| = \frac{9 \cdot 8 \cdot 7}{6} = 84. \]
Find the smallest \( n \) such that the probability that these two squares are adjacent (horizontally or vertically) is \( \frac{1}{2015} \).

**Step 1:** For fixed \( n \) calculate the probability that two randomly chosen squares are adjacent.

**Step 2:** Find the smallest \( n \) such that the probability from step 1 is \( \frac{1}{2015} \).

**Step 0:** What is the sample space \( S \)?

### Evaluation of \( |E_H| \)

\[
|E_H| = |E_H^1| + |E_H^2| + |E_H^3| + \ldots + |E_H^m|
\]

\( |E_H^k| = \# \) of ways to pick two adjacent squares in the \( k \)-th row.

The left square can be chosen in \( (m-k) \) ways, the right one is uniquely determined.

\[
|E_H^k| = 2(n-1) \Rightarrow |E_H| = n \cdot 2(n-1)
\]

### \( n(n+1) > 4 \cdot 2015 \)

\[
\Rightarrow 2^{10} = 1024 \quad 32^2 = 1024
\]

If you try \( n = 89 \), \( 90 \), \( 89 + 89 < 4 \cdot 2015 \) \( \approx 2 \cdot \sqrt{2} \cdot 32 = 1.9 \cdot 64 \)

### \( 64 \cdot 1.4 = 89.6 \)

\( 64 \)
No matter what the first square was, there are 4 adjacent squares next to it.
7 dwarfs. Each gets an R or G hat. At the same time, each dwarf has to say "R," "G," or keep silent.

They go free if all of those who talked correctly said the color of their hat. If one or more of them said a color incorrectly, they all lose. If they are all silent, they all lose.

Is there a good strategy?

3 Dwarfs

\[ S = \{RRR, RRG, RGR, RGG, GRR, GRG, GGR, GGG\} \]

Strategy: If a dwarf sees two friends having the same color hats, the dwarf talks and tells the opposite color.

If the configuration is RRG

Dwarf 1: "Green"
Dwarf 2: "Green"
Dwarf 3: "Green"

Die with a glory.

If one or more of them said the color of their hat incorrectly, they all lose. If they are all silent, they all lose.

\[ P(\text{Survival}) = \frac{6}{8} = 75\% \]

With 3 dwarfs, there is one strategy that looks good. One dwarf is chosen to be special. That special dwarf is the only one who will guess. The other two remain silent. The probability of survival is 50%.

Assume you are one of the dwarfs. If you see two friends with the same color, that means the chances of survival are 50%. If you see two friends with different colors, the survival chance is 100%.
The standard deck of 52 cards is properly shuffled. Cards are flipped one by one. You start with $100. Before each flip you can bet on the next card. If you bet $x and you guess correctly, you end up with $2x. If you are wrong, you lose $x. What is the maximal amount of money that you are guaranteed to win?

We can make $200 wait until the last card that you know for sure what it will be and bet all the money on that card.

Hint: Start with a deck of 4
Cards: 2R and 2B

4. $(100-x)=2(100+x) \iff
400-4x=200+2x\iff
200=6x\iff x=\frac{200}{6}=\frac{100}{3}.

There is a strategy that guarantees that we will make
\[2 \cdot \left(100 + \frac{100}{3}\right) = 2 \cdot \frac{400}{3} = \frac{800}{3} = 266.66\]