26.4. Take-Home Problem Set

Problem 1. Our office has 9 computers. My manager wants to link them into a network so that every computer would be connected with exactly 4 computers by network cables.

    (a) How many cables will we need? Can you draw such a configuration?

    (b) The manager decided to connect every computer with exactly 5 other computers. Is this a possible configuration?

Problem 2. There are 15 planets that circle the star called Aldebaran. Some of these planets have signed trade agreements with each other. Could it be possible that:

    - 4 of these planets have exactly 4 trade partners each,

    - 8 of these planets have exactly 5 trade partners each,

    - the rest of these planets have exactly 3 trade partners each?

Problem 3. During a chess tournament some people played 5 games and some people played 6. Prove that the number of people who played 5 games is even.

Problem 4. Eight people came to a party, some of them shook hands. Is it possible that 6 of them shook hands with 6 different people at the party, and 2 shook hands with only 2 each?

Problem 5. Martian amoebas gemmate (they multiply by division). When a red amoeba divides, it splits itself into 5 blue amoebas; when a blue amoeba divides, it splits into 7 red amoebas. When my spaceship left Mars, I put 1 amoeba into a jar. Upon arriving on Earth, the customs officer found 100 amoebas in that jar. Why does he keep searching my ship, insisting that some amoebas must have escaped?

Problem 6. The 99 greatest scientists of Mars and Venus are seated evenly around a circular table. If any scientist sees 2 colleagues from his own planet sitting an equal number of seats to his left and right, he waves to them. For example, if you are from Mars and the scientists sitting 2 seats to your left and right are also from Mars, you will wave to them. Prove that at least 1 of the 99 scientists will be waving, no matter how they are seated around the table.