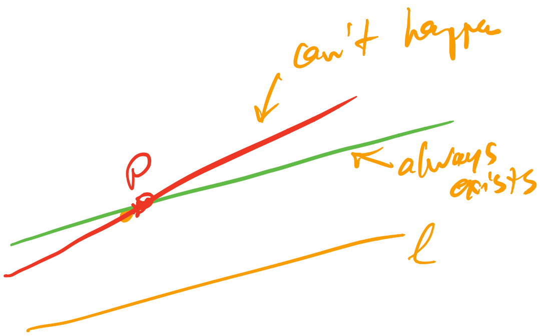
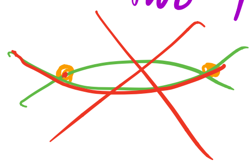
How many points are on a plane?

Euclid's axioms

(1) parallels ?

(2) a line is defined by two points





- can't have 2 different lines through two pts
- and one always exists

Definition A **plane** is a set, with elements called **points**, with special subsets called **lines** which satisfy the following properties.

A1. (Incidence Axiom) Every two *distinct* points belong to a *unique* line.

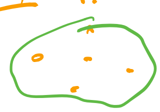
A2. (Paralles Axiom) For every line and a point *not on it* there is a *unique* line containing this point parallel to the given line.

i.e. have no common pts

A3. (Dimension Axiom) There exist 3 points which are not *collinear*.

i.e. ~~not~~ on same line

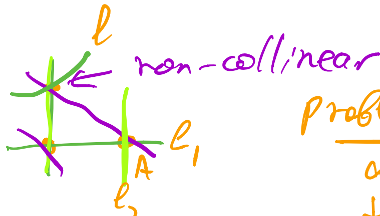
Ex 1.



- 5 pts, 1 line

not a plane b.c. it fails dim A3

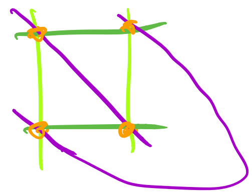
Ex2



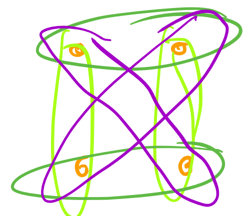
problem: lines ℓ_1 and ℓ_2 are ∥ to ℓ and pass through A

So A2 uniqueness doesn't hold.

Ex 3



4 pts
6 lines

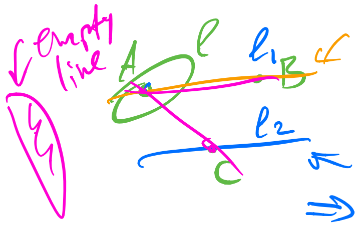


← It is indeed, a plane!
(smallest that exists)

So 4 pts can be on a plane

Th 1) Every line has at least 2 pts

Pf Assume that ℓ has only 1 pt A



← exists by A1

$\ell_1 = AB$

$\ell_2 \parallel \ell_1$ ℓ_2 contains C

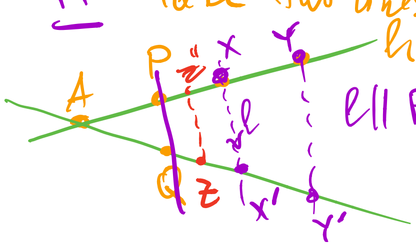
$\Rightarrow$ Both ℓ and ℓ_1 are $\parallel \ell_2$ and pass through A

$\Rightarrow$ violation of uniqueness in A2

Similarly, no empty lines $\Rightarrow$ all lines must have at least 2 pts.

Th 2) All lines have same number of pts.

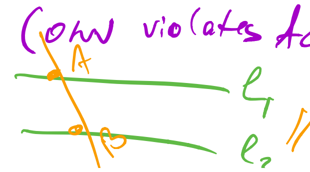
Pf Take two lines ℓ_1 and ℓ_2, assume ℓ_1 and ℓ_2 pass A



$\ell \parallel PQ$ ℓ passes through X on ℓ_1

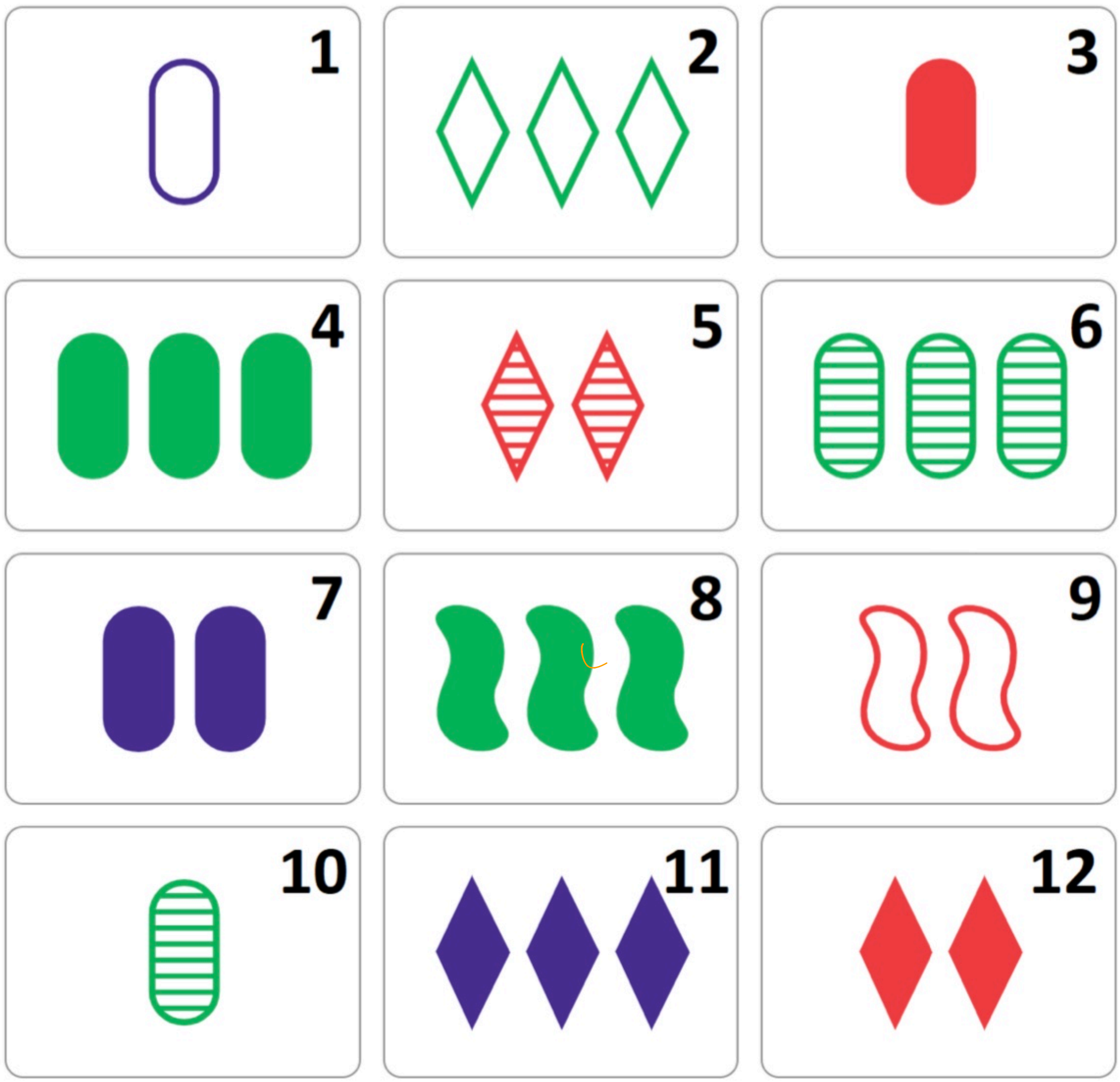
then ℓ intersects ℓ_2 (o/w violates A2)

If $\ell_1 \parallel \ell_2$



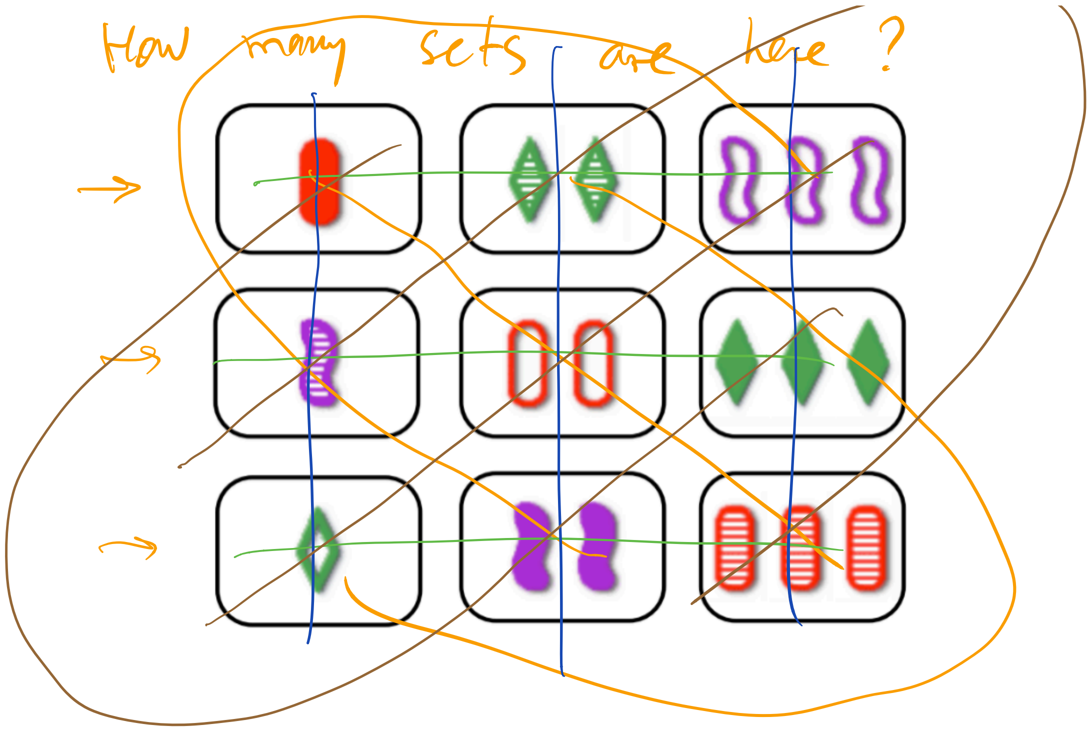
How many points are on a plane?

(1,3,10) (9,10,11) (1 5 8) (3 4 7) (2 6 8) (1,2,9)



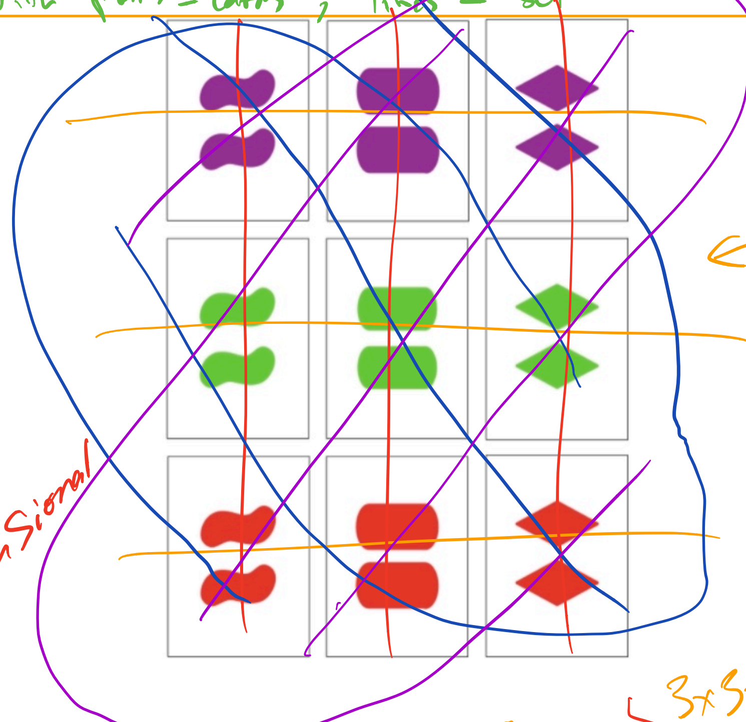
Fact Any two "pts" (cards) belong to a unique set.

How many sets are here ?



12 sets Why no more ?

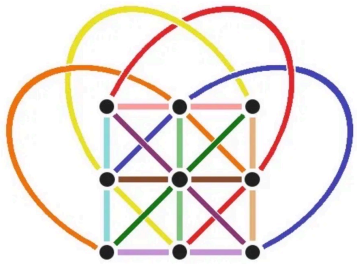
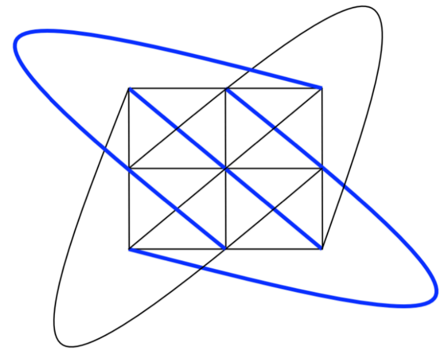
And in fact this is a 9 pt plane
with points = cards ; lines = sets



Baby Set with only 2 "variables" left
i.e. it's 2-dimensional

← 3x3 plane

So the big set is a 4-dim space
3x3x3x3

Questions:

1. Can there be a plane w 5, 6, 7, 8 points?
(No! But why?)

2. Show that if a line has n pts, then the plane has n^2?

3. Can we have 16 or 25 pts?

4. How about 36?