Int II Invariants

Wednesday, October 28, 2020 8:45 PM

33 squares

5

5

33 squares

would need \(33 \times \frac{1}{2} = 16.5\) dominoes

Impossible

24 white squares

22 purple squares

Impossible

\[
\frac{68 - 2}{2} = 23\]
dominos

cover 23 white

Possible \( \checkmark \) 23 purple

50 red

58 white

no contradiction

26 red

26 white

24 green

24 purple

Tetromino will cover 1 of each color.

\( \rightarrow 25 \text{ red/14 purple} \)

52 red each tetromino covers 2 red/2 white

48 white

10 red

10 white

4

5

Tetromino

10 red/11 white

9 red/11 white

11 red/19 white

25 black squares

24 white squares

black \( \rightarrow \) white

11
Invariants 1

BMC Int II Fall 2020
October 28, 2020

1 Colorings Warm Up

1. We want to use dominoes to tile an \( m \times n \) board whose two diagonally opposite corner squares have been removed. Is this possible for a \( 8 \times 8 \) board? A \( 3 \times 7 \) board? What about a \( 6 \times 7 \)
1. Colorings Warm Up

1. We want to use dominoes to tile an $m \times n$ board whose two diagonally opposite corner squares have been removed. Is it possible for a $6 \times 6$ board? A $5 \times 7$ board? What about a $6 \times 7$ board?

2. Can an $8 \times 8$ chessboard with the squares C6 and G2 cut out be tiled with dominoes?

3. Prove that a $10 \times 10$ board cannot be tiled with $1 \times 4$ trominoes.

4. Can we cover a $4 \times 5$ rectangle with 5 tetris figures?

5. A class of 10 students have their desks arranged in a $7 \times 7$ square. No more than one student can fit in a single desk. The teacher asks every student to move to an adjacent desk (directly in front, behind, to the left, or to the right). Is it possible to fulfill the teacher’s wish?

6. Can we tile an $8 \times 8$ board with $32 \times 1 \times 2$ dominoes so that 17 of them are horizontal and 15 are vertical?

7. Prove that a $50 \times 50$ board cannot be tiled with $T$-shaped tetrominoes.

8. A mouse is at a corner of a $5 \times 7$ board with its center square missing. Is it possible for the mouse to visit every other square on the board without visiting any square twice?

9. Now the mouse is at the corner of an $8 \times 8$ chessboard. Is it possible for the mouse to visit every other square on the board without visiting any square twice, so that the mouse ends up at the opposite corner of the chessboard?

2. More Difficult Colorings

1. A $3 \times 3 \times 3$ watermelon cube and consists of 27 smaller cubical watermelon pieces arranged in a $3 \times 3 \times 3$ pattern. A caterpillar is eating the watermelon one small piece at a time and then moving to an adjacent piece. Is it possible for the caterpillar to start at one corner and then finish at the central piece? (The watermelon is floating in water so the pieces will not fall down even if the pieces underneath are eaten.)

2. Prove that a $5 \times 13$ rectangle cannot be tiled by L-shaped tetrominoes.

3. Can we tile an $8 \times 8$ chessboard with $1 \times 3$ trominoes so that only one corner is uncovered?

4. We tile an $8 \times 8$ chessboard with $1 \times 3$ trominoes so that one square is uncovered. What are the possibilities for this uncovered square?

5. Is it possible to tile a $13 \times 13$ board with $1 \times 2$ dominoes only placed horizontally and $1 \times 3$ tromino only placed vertically?

6. Prove that we cannot tile a $6 \times 6$ square with $1 \times 4$ trominoes.

7. A rectangular board was originally tiled by $1 \times 4$ tetrominoes and $2 \times 2$ squares. All the pieces were then removed from the board but one $2 \times 2$ square was lost in the process. Instead, an additional $1 \times 4$ tetromino was provided. Prove that the board cannot be tiled with these new pieces.

8. A $7 \times 9$ rectangle is tiled with L-shaped tetrominoes (4 squares) and $2 \times 2$ squares. How many $2 \times 2$ squares could we use?

9. We tile a $6 \times 6$ board with dominoes. Prove that we can use a vertical or horizontal line to divide the board into two parts without cutting any domino.
Prime in any tiling, one of the 5 vertical or 5 horizontal lines will not bisect a domino.

Assume for contradiction we could tile a board so each line bisects at least one domino.

Can it bisect only 1 domino?

No, must bisect on even # of dominoes.

Each line must bisect at least 2 dominoes.

Each domino is bisected by at most 1 line.

10 lines $\Rightarrow \geq 20$ dominoes must be bisected

But only 18 dominoes total.