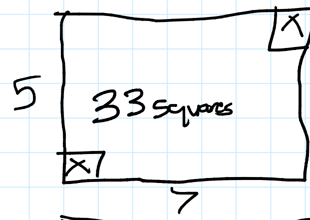
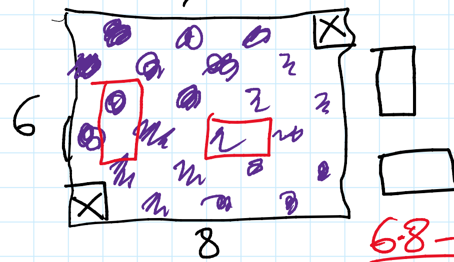
Int II Invariants

Wednesday, October 28, 2020 8:45 PM



⇒ would need $33/2 = 16\frac{1}{2}$ dominos.

Impossible



24 white squares

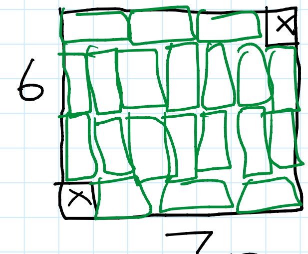
22 purple squares

Impossible

$\frac{6 \cdot 8 - 2}{2} = 23$ dominos

cover 23 white

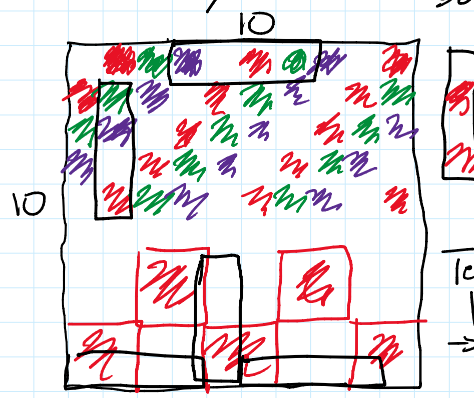
23 purple



Possible ✓

50 red
50 white

no contradiction



26 red

26 white

24 green

24 purple

Tetromino will cover 1 of each color.

→ 25 r/w/g/p

52 red

48 white

each tetromino covers 2 red 2 white

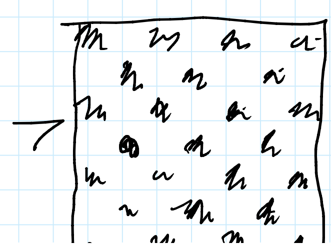
10 red

10 white

Adding:

9 red/11 white

11 red/9 white

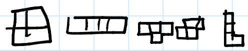


25 black squares

2~~4~~ white squares

black → white

white → black



same color.

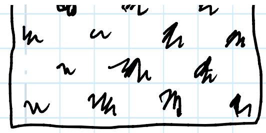
No

No

No

No

start→

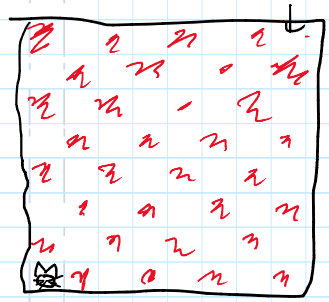


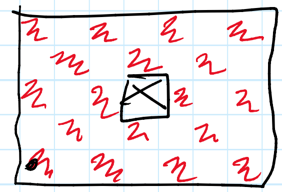
black → white

white → black

7. 25 people ⇒ 24 desks

Not possible.





Path: w→r→w→r...

Hit every square, 6 9th → red

34

18 red

16 w

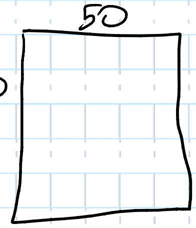
rwrw ... rw

17 red

17 white

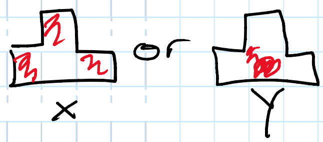
50

50



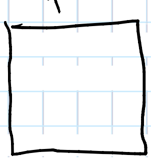
checkerboard coloring → $25 \cdot 50 = 1250$ red

1250 white



4

4



$$x + y = \frac{50 \cdot 50}{4} = 625$$

$$\#\text{red squares} = 1250 = 3x + 1y = 3x + y$$

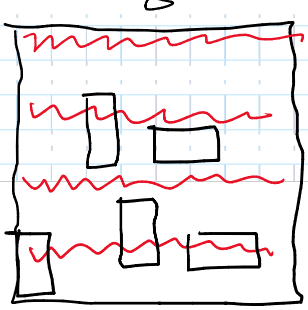
$$3x + y = 1250$$

$$\underline{x + y = 625}$$

$$2x = 625 \rightarrow \text{contradiction}$$

8

8



17 vertical → 17 red, 17 white

15 horizontal ? → even # red, even white

odd red, odd white Not possible.

32 red

32 white

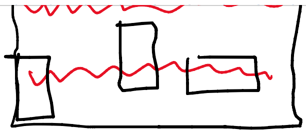
Invariants I

BMC Int II Fall 2020

October 28, 2020

1 Colorings Warm Up

1. We want to use dominoes to tile an $m \times n$ board whose two diagonally opposite corner squares have been removed. Is this possible for a 6×8 board? A 5×7 board? What about a 6×7



32 red
32 white

odd red, odd white Not possible.

Invariants I

BMC Int II Fall 2020

October 28, 2020

1 Colorings Warm Up

1. We want to use dominoes to tile an $m \times n$ board whose two diagonally opposite corner squares have been removed. Is this possible for a 6×8 board? A 5×7 board? What about a 6×7 board?
2. Can an 8×8 chessboard with the squares $C6$ and $G2$ cut out be tiled with dominoes?
3. Prove that a 10×10 board cannot be tiled with 1×4 tetrominos.
4. Can we cover a 4×5 rectangle with 5 tetris figures?
5. A class of 49 students have their desks arranged in a 7×7 square. No more than one student can fit in a single desk. The teacher asks every student to move to an adjacent desk (directly in front, behind, to the left, or to the right). Is it possible to fulfill the teacher's wish?
6. Can we tile an 8×8 board with 32 1×2 dominoes so that 17 of them are horizontal and 15 are vertical?
7. Prove that a 50×50 board cannot be tiled with T-shaped tetrominos.
8. A mouse is at a corner of a 5×7 board with its center square missing. Is it possible for the mouse to visit every other square on the board without visiting any square twice?
9. Now the mouse is at the corner of an 8×8 chessboard. Is it possible for the mouse to visit every other square on the board, without visiting any square twice, so that the mouse ends up at the opposite corner of the chessboard?

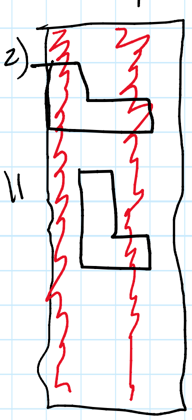
2 More Difficult Colorings

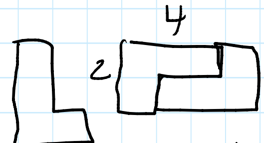
1. A $3 \times 3 \times 3$ watermelon cube and consists of 27 smaller cubical watermelon pieces arranged in a $3 \times 3 \times 3$ pattern. A caterpillar is eating the watermelon one small piece at a time and then moving to an adjacent piece. Is it possible for the caterpillar to start at one corner and

then finish at the central piece? (The watermelon is floating in water so the pieces will not fall down even if the pieces underneath are eaten) No

2. Prove that a 4×11 rectangle cannot be tiled by L-shaped tetrominoes.
3. Can we tile an 8×8 chessboard with 1×3 trominos so that only one corner is uncovered? No
4. We tile an 8×8 chessboard with 1×3 trominos so that one square is uncovered. What are the possibilities for this uncovered square?
5. Is it possible to tile a 13×13 board with 1×2 dominos only placed horizontally and 1×3 trominos only placed vertically?
6. Prove that we cannot tile a 6×6 square with 1×4 tetrominos.
7. A rectangular board was originally tiled by 1×4 tetrominos and 2×2 squares. All the pieces were then removed from the board but one 2×2 square was lost in the process. Instead, an additional 1×4 tetromino was provided. Prove that the board cannot be tiled with these new pieces. 4 colors
8. A 7×9 rectangle is tiled with L-shaped trominos (3 squares) and 2×2 squares. How many 2×2 squares could we use? rows
9. We tile a 6×6 board with dominos. Prove that we can use a vertical or horizontal line to divide the board into two parts without cutting any domino.

13 No

2) 

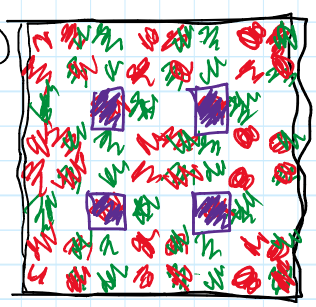


3 red/1 white x
or 1 red/3 white y

$x+y=11$

$3x+y=\#\text{red}=22$

$\rightarrow 2x=11 \rightarrow$ ×

3) 

1 rgw

Must remove red cell

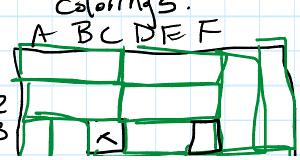
21 white
22 red
21 green

equal # of rgw cells &

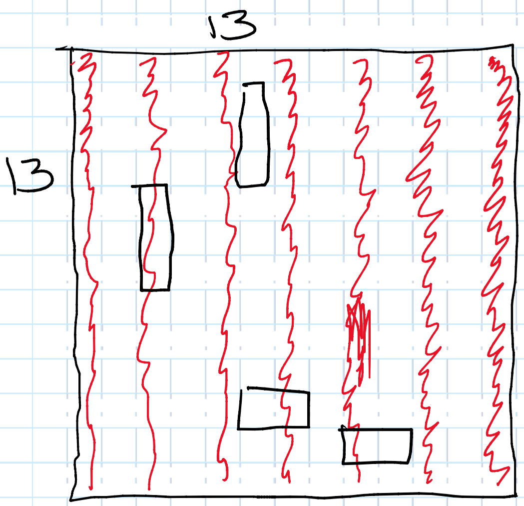
only candidates

red under both colorings.

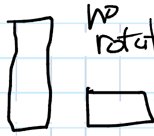
Must remove red cell



It is possible to tile if we remove C3. By symmetry (rotation)



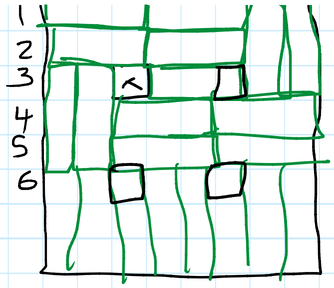
No



no rotation

78 white

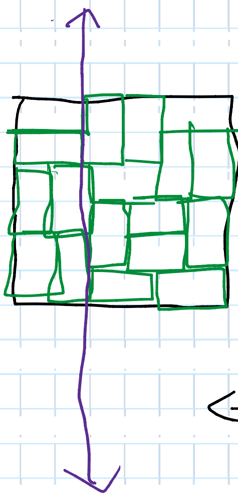
91 red



It is possible to tile if we remove C3.
By symmetry (rotation) F3, C6, F6 are also possible.

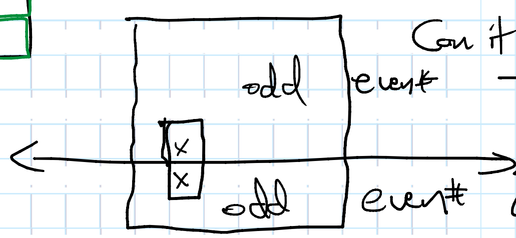
$\#\text{red} - \#\text{white} = 13$

made up from 3×1 trominos must be a multiple of 3 ↯



Prove in any tiling, one of the 5 vertical or 5 horizontal lines will not bisect a domino.

Assume for contradiction we could tile a board so each line bisects at least one domino.



Can it bisect only 1 domino?
→ No, must bisect an even # of dominos.

Each line must bisect at least 2 dominos.
Each domino is bisected by at most 1 line.

10 lines → ≥ 20 dominos must be bisected

But only 18 dominos total ↯