Berkeley Math Circle

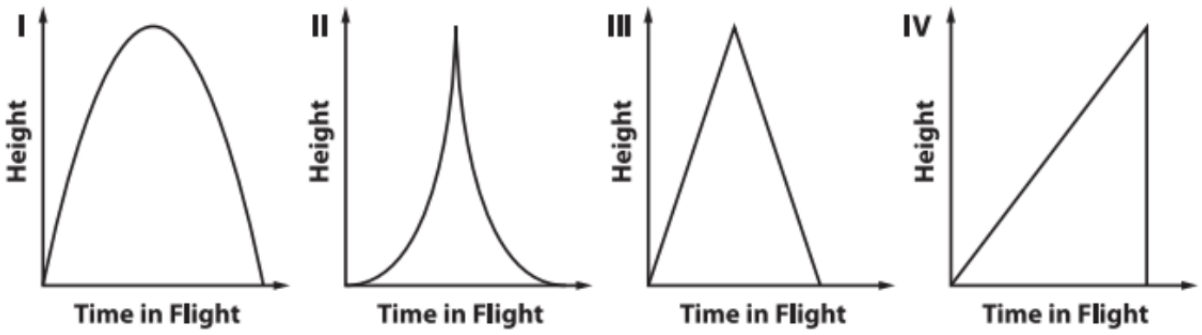
Quadratics and Area

October 2020

Think about the situation

The highest pumpkin chuck ever recorded happened in 2010, in Moab Utah at a height of 5,545.43 ft (that's over one mile high!!). The pumpkin was shot by a pneumatic cannon.

a Which of these graphs is most likely to fit the pattern relating pumpkin height to time in flight? Explain your choice.



b What pattern would you expect to find in data tables relating pumpkin height to elapsed time?

PART I

1. Use an area model to multiply $(6x - 1)(3x + 2)$. Write your solution, S, as a sum.

2. Find the sum and the product for the following area models. Are there any specific strategies you discovered that can help you determine the dimensions of the rectangle? Be prepared to discuss these strategies.

a.

$2x$	5
$6x^2$	$15x$

b.

$-2y$	-6
$5xy$	$15x$

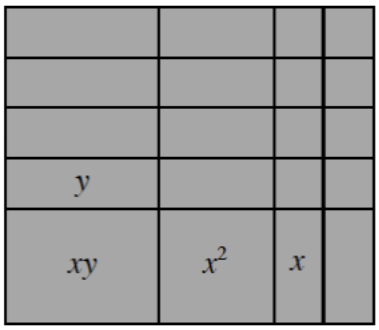
c.

$-9x$	-12
$12x^2$	$16x$

3. Write the area of the rectangle below as a sum and a product.

$-3x$	$-6y$	12
$2x^2$	$4xy$	$-8x$

4. Write the area of the rectangle below as a product and as a sum.



PART II

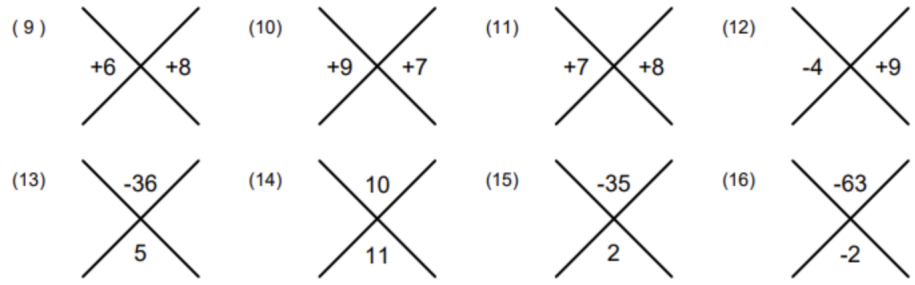
1. Multiply the expressions below using an area model. Then verify Casey's pattern (product of one of the diagonals = the product of the other diagonal)

   (a) $(4x - 1)(3x + 5)$

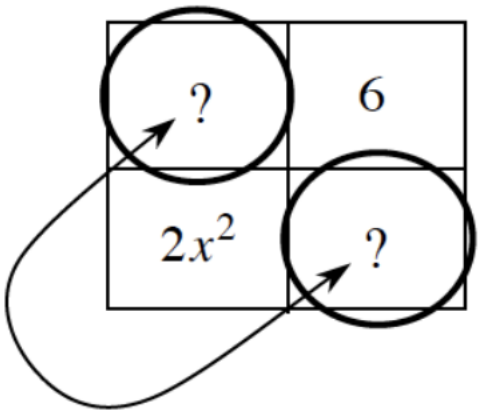
   (b) $(2x - 7)^2$

2. See directions below.

Complete the diamond problems. The top cell contains the *product* of the numbers in the left and right cells, while the bottom cell contains the *sum*.



3. Kelly wants to find a shortcut to factor $2x^2 + 7x + 6$. She knows that $2x^2$ and 6 go into the rectangle in the locations shown below. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle?



4. To complete Kelly's generic rectangle, you need two x-terms that have a sum of $7x$ and a product of $12x^2$. Create and solve a Diamond Problem that represents this situation.

5. *The process of changing area as a sum to area as a product is called* ***factoring***. Use the process we just developed in problem 4 to factor the following quadratics if possible.

   (a) $x^2 - 4x - 12$

   (b) $4x^2 + 4x + 1$

(c) $2x^2 - 9x - 5$

(d) $3x^2 + 10x - 8$

6. For each problem, write the area as a sum and as a product. If you cannot build a rectangle, be prepared to convince others that no such rectangle exists.

(a) $x^2 + 7x + 6$

(b) $x^2 + 2x + 3$

(c) $x^2 + 2x$

(d) $x^2 + 7x + 12$

(e) $x^2 + 7x + 9$

PART III

1. The multiplication table below has factors along the top row and left column. Their product is where the row and column intersect. Complete the table with all of the factors and products.

Multiply	$x - 2$	
$x + 7$		
	$3x^2 - 5x - 2$	$6x^2 + 5x + 1$

2. Prove the following identities using an area model to express the product.

   (a) $(a + b)^2 = a^2 + 2ab + b^2$

   (b) $(a - b)^2 = a^2 - 2ab + b^2$

   (c) $(a + b)(a - b) = a^2 - b^2$

3. Factor each expression completely, if possible. Don't be afraid to use formulas for quick multiplication.

   (a) $9x^2 - 25$

   (b) $x^2 + 18x + 81$

   (c) $3x^2 - 24xy + 48y^2$

   (d) $x^2 - 4x + 4$

PART IV

1. Solve each equation by factoring

   (a) $x^2 - 13x = 30$

   (b) $27x^2 = 12$

   (c) $(x + 3)^2 - (2x - 1)^2 = 0$ (Hint: use difference of squares)

2. You are managing a 1600 unit apartment complex. Currently, 800 units are rented at \$300/month. A market survey has indicated that for each \$5 you decrease the rent you will get by 20 new leases.

   (a) Determine the function R(x) that models the total rental income, where x is the number of \$5 decreases in monthly rent.

   (b) Maximize the revenue for this apartment building (aka; find the vertex).

3. When running a business, your profit = revenue - cost $P(x) = R(x) - C(x)$ Your local newspaper, The East Bay Times, has fixed production costs of \$70 per edition, and marginal printing and distribution costs of 40cents/copy. The East Bay Times sells for 50 cents/copy.

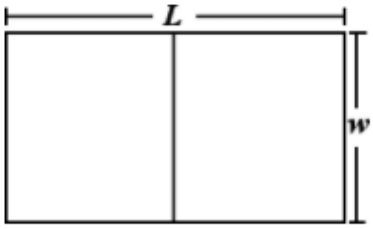
   (a) Write down the associated cost, revenue, and profit functions.

   (b) What profit (or loss) results from the sale of 500 copies of The East Bay Times?

   (c) How many copies should be sold in order to break even? (Revenue = Cost)

4. You have a 500-foot roll of fencing and a large field. You want to construct a rectangular playground area. What are the dimensions of the largest such yard? What is the largest area?

5. You have a 1200-foot roll of fencing and a large field. You want to make two paddocks by splitting a rectangular enclosure in half. What are the dimensions of the largest such enclosure?



6. Your factory produces lemon-scented widgets. You know that each unit is cheaper, the more you produce. But you also know that costs will eventually go up if you make too many widgets, due to the costs of storage of the overstock. The guy in accounting says that your cost for producing x thousands of units a day can be approximated by the formula $C = 0.04x^2 - 8.504x + 25302$. Find the daily production level that will minimize your costs.